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A  
P L A N  
OF A  
COURSE OF LECTURES  
ON THE  
P R I N C I P L E S  
OF  
NATURAL PHILOSOPHY.

BY THE  
Rev. S. VINCE, A.M.F.R.S.

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C A M B R I D G E

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A

N A L P

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ON THE

PRINCIPLES OF

NATURAL PHILOSOPHY



OF THE NATIONAL ACADEMY OF SCIENCES

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THE NATIONAL ACADEMY OF SCIENCES  
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**T**HE following Plan of a Course of Lectures on the Principles of Natural Philosophy was drawn up in consequence of the author's appointment to read the *Plumian Lectures* on *Experimental Philosophy* and *Practical Astronomy*. Some propositions are added, besides those which can be subjected to experimental proof, in order to preserve their connection, that it might at the same time direct the theoretical student. Such observations are added as were judged might be useful, both to illustrate the experiments and assist the reader.

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plan to illustrate the experiments and to the

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# M E C H A N I C S.

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## ON ATTRACTION AND REPULSION.

Pr. 1. **A**LL bodies upon the earth's surface have a tendency to move towards it.

This tendency of bodies towards the earth arises from the compound tendency towards all its parts. For by an experiment made by Dr. MASKELYNE upon the side of the mountain Schehallian, he found that a pendulum had a tendency towards the mountain. Allowing therefore the tendency which all bodies upon the earth have to every part of the earth, it will follow that every part of the earth has a like tendency to every body upon the earth. For if *A* have a tendency to *B*, and *A* and *B* be matter of the same kind, *B* must have a like tendency to *A*. The tendency of the pendulum towards the mountain was observed to be much less than the tendency of a body towards the whole earth. The tendency therefore of *A* towards *B* appears to be greater the greater *B* is. A small body will therefore have a greater tendency towards a large one, than a large body has towards a small one. This power by which bodies are thus made to approach each other is called attraction of GRAVITATION.

If a glass bubble be placed upon water in a vessel near to its side, it is observed to move up to the side. Or if two light bodies be laid upon water near to each other, they will move up to each other, and the smallest body will move the fastest. These have been brought as instances of the attraction of the bubble to the side of the vessel, and of the mutual attraction of the bodies towards each other; but the motions which they are observed to have are much greater than what could arise from the attraction of gravitation. If a cork be laid upon water, and another cork be holden near to it without touching the water, it will produce no effect; but as soon as the cork is put into the water the other cork moves up to it. They do not come together therefore by their mutual attraction.



2. If two glass planes be moistened with oil and inclined at a very small angle, and a drop of oil be put between them, it will move towards their concurrence.

This proves that there is an attraction between the glass and the oil. The attraction which is observed to take place between two bodies in contact, is called the attraction of COHESION.

3. If two globules of quicksilver lying on a smooth plane be brought to touch, they immediately rush together and form one globule.

This can arise only from their strong attraction. Drops of water will do the same.

4. Water is attracted by glass.

This appears from its rising at the edge of a glass vessel above the level of the other part. Also if very small glass tubes be dipped in water, the water will stand in them above the level of the water in the vessel. Or if two glass planes forming a small angle be immersed perpendicularly in water, the water rises between them above the surface in the vessel, and the height is greater the nearer to the concurrence of the planes. The curve terminating the top of the water between the planes is an hyperbola.

5. Quicksilver is attracted by glass.

For a globule of quicksilver will adhere to the under side of a clean piece of glass. Its attraction therefore to the glass must be greater than its gravity.

6. The particles of water are more strongly attracted by glass than by each other.

This appears from the rising of the water in small glass tubes, and between the glass planes, above the level of the water in the vessel.

7. The particles of quicksilver attract each other more than they are attracted by glass.

For if a glass tube be put into quicksilver, the quicksilver will not rise in it so high as the surface of the quicksilver in the vessel. Also if two glass planes be put perpendicularly into quicksilver, and be inclined at a small angle, the quicksilver will stand at a less height between them than that of the surface of the fluid in the vessel. The curve terminating the surface is an hyperbola.

8. Mercury

8. Mercury attracts silver more strongly than it does lead, lead more strongly than brass and brass more strongly than steel.

It is remarkable that the chemical affinities of these bodies to mercury follow the same order.

9. If two polished plane surfaces of metal, &c. be besmeared with oil, grease, &c. and pressed together, they are observed to cohere very strongly.

The oil fills up the cavities and dislodges the air, which prevents the attraction of cohesion from taking place. If the air be expelled by different substances, as oil, turpentine, grease, &c. it is found that the attraction of cohesion is different. These substances therefore tend, in some way or other, to affect the attraction. It is this attraction of cohesion by which the parts of a body are kept together. When you break a body you only overcome this attraction, and could you join again the parts exactly in the same manner, it would be as strong as it was before. The soldering of metals, glueing of bodies, &c. is only the bringing of the constituent particles so near that the attraction of cohesion may take place. The solder and the glue attach themselves to each body upon this principle, and thus connect the bodies. It seems as if a contact of the parts must take place before this effect can be produced, for if the body be not well cleaned the solder will not hold them so firmly together.

Hence we may explain why some bodies are hard, others soft, others fluid, &c. Hard bodies may consist of particles which touch in a great part of their surfaces, and thus they will have a great power of attraction. The constituent particles of soft bodies may touch in a less quantity of surface, by which the attraction will be weakened. And fluids may consist of globular particles, which touching each other only in one point, their attraction must be so weak as to permit them to move with the utmost facility amongst each other. Elasticity may arise from the particles of a body when disturbed not being drawn out of each others attraction; as soon therefore as the force upon it ceases to act, they restore themselves to their former position.

Solids are dissolved in menstrooms from the particles of the solid being more attracted to the fluid than to themselves. And precipitation arises from the same cause; for if to the solution of any solid in a fluid, some other solid or fluid be added whose particles are attracted by the fluid with a greater force than those of the solid which was dissolved, that solid becomes disengaged and falls to the bottom in a fine powder. Thus silver dissolved in aqua fortis is precipitated by copper.

10. If several bodies, as glass, amber, sealing wax, &c,

wax, &c. be rubbed with dry woollen cloth, they will both attract and repel light bodies.

This is called ELECTRICAL attraction and repulsion.

11. The same poles of two magnets repel, and the contrary poles attract each other.

This is called MAGNETIC attraction and repulsion.

The existence of an elastic fluid proves that the particles must be kept at a distance by a repulsive force. The difficulty of mixing many fluids, as oil and water, probably arises from the repulsion between the particles. That metals when dissolved in fluids should diffuse themselves equally through the fluid, has been supposed to be owing to a repulsive force, which takes place after the particles are separated to a certain distance.

From the principle of attraction and repulsion Sir I. NEWTON accounts for all the phenomena of nature. By the attraction of gravitation he accounts for the motions of the heavenly bodies; and for the motions between the component particles of bodies by attraction and repulsion. But why the particles should in some cases attract and in others repel, he attempts not to explain. See his Optics, Qu. 31.

#### L A W S O F M O T I O N.

12. Every body perseveres in its state of rest or uniform motion in a right line, until some external force acts upon it.

For no effect can be produced without a cause, and no cause is here supposed to act, the body being supposed to be void of self motion. The force with which a body resists any change, is called its *VIS INERTIÆ*.

13. The change of motion is in proportion to the force impressed, and takes place in the direction in which the force acts.

The first part of this is only measuring the effect by the cause, supposed to act for the same time; and the second part is manifest, it being evident that the effect of the force must take place in the line in which it acts.

14. Action and reaction are equal and contrary.

The meaning of this is, that when two bodies move in opposite directions and strike each other, they lose equal quantities of motion in their



their respective directions, measuring the motion by the velocity and quantity of matter conjointly; and if they move in the same direction, the quantity of motion which the striking body loses in that direction, the other gains. Hence the quantity of motion in the same direction is not altered from the collision of bodies.

This may be proved either directly by experiments, or by assuming the principle and showing that the theory of the collision of bodies agrees in all cases with the experiments. Sir. I. NEWTON established its truth by experiments of the latter kind, by suspending two elastic balls and letting one descend in a circular arc and strike the other; and by estimating the velocity of the striking body before impact and of each after, he found the law to obtain. See the PRINCIPIA, Scholium to the Laws of Motion.

These three laws of motion are assumed by Sir I. NEWTON as the fundamental principles of mechanics; and the theory of all motions deduced from them, as principles, being found to agree in all cases with experiments and observations where they can be applied, these laws are considered as mathematically true.

#### ON THE COMPOSITION AND RESOLUTION OF MOTION, AND THE MECHANICAL POWERS.

15. If a body be in motion and another body be projected from it, the latter body, besides its projectile motion, will retain the motion of the former body.

It was at first objected to the earth's motion, that in that case a stone thrown perpendicularly upwards ought not to fall down again in the same place, but to be left behind. But the contrary admits of a very satisfactory proof by experiment.

16. If a body be acted upon by two single impulses, or by any two continued forces each of which always acts parallel to itself, it will not by one of the motions *A* be hindered by the other *B* from approaching a line parallel to the direction in which *B* takes place.

Upon this principle depend the composition of motion and the doctrine of projectiles.

17. If a body be acted upon by any two single im-

impulses, it will describe the diagonal of a parallelogram in the same time it would have described either side, had the forces acted separately.

*See pra. 163.*

18. If a body be kept at rest by three forces, and in the directions in which they act lines be drawn from the body proportional to them, and any two of these lines be completed into a parallelogram, the diagonal will be equal and opposite to the third line.

Hence any two forces may be compounded into one which shall in every respect be equivalent to them; and therefore conversely, any single force may be resolved into two in any two directions, by describing upon the line representing the single force a parallelogram whose sides shall lie in the required directions.

19. When a body is kept at rest by three forces, they will be as the three sides of a triangle parallel to the directions in which they act.

For they are represented by the diagonal and two sides of the parallelogram forming a triangle, two of whose sides lie in the direction of two of the forces, and the third side parallel to the third force, and if lines be drawn parallel to these directions they will form a similar triangle, and therefore the proportion of the sides will be the same. It follows also that the three forces will be as the respective sides of a triangle perpendicular to which they act, because such a triangle will be similar to the other. The three forces must be all directed to the same point, otherwise they will give the body a rotatory motion.

Cor. Hence the converse is true, that if a body be acted upon by three forces proportional to the three sides of a triangle parallel to which they act, it will be at rest.

20. If a body  $B$  rest upon a string, between two pulleys in an horizontal position, being balanced by two equal bodies  $A$ ,  $A$  hanging on opposite sides, the distance of  $B$  from the horizontal line joining the pulleys will be  $\frac{B d}{\sqrt{4 A^2 - B^2}}$ ,  $d$  being half the distance of the pulleys.

If  $d = 12$  in.  $A = 4$  oz.  $B = 3$  oz. the distance of  $B$  from the horizontal line  $= 4.85$  in.

21. If the power and resistance act perpendicular to the sides of a wedge at rest, they will be as the three respective sides.

This follows from the observation to proposition 19, by substituting a triangle for the body, and conceiving the three forces to act perpendicular to the three sides.

By the resolution of motion, the effect of any force oblique to a plane in a direction perpendicular to it, varies as the force  $\times$  sin. Incl. therefore if the power  $P$  and resistances  $R, R'$ , act obliquely to the sides of the wedge at the angles  $p, r, r'$  respectively, the back and respective sides will be as  $P \times \sin. p, R \times \sin. r$  and  $R' \times \sin. r'$ . It is here supposed that the part of the force acting parallel to the side is all lost on account of the obliquity at which the forces act, which can only hold upon supposition that there is no friction. *see pa. 163.*

When a wedge is driven into a piece of wood, the friction is greater than the power necessary to preserve the equilibrium; for when the power ceases to act at the back, the friction prevents the resistances from driving it out, which they necessarily would when the power was removed, were it not for the friction. In like manner nails, pegs, &c. are retained by friction, for being made tapering, the effect of the resistance of the wood is to drive them out.

In the estimation of the proportion of the power to the weight in all the mechanic powers, there is supposed to be an equilibrium between them.

22. If any two weights balance each other when hung upon a straight lever, they will be to each other inversely as their distances from the fulcrum.

Hence when the distances are equal the weights are equal, which is the case with the common scales.

If the arms  $a, b$ , are not of an equal length, the true weight  $v$  will be a geometrical mean between the weights  $m$  and  $n$  which will balance it when hung first on one end and then on the other; for let

$$\left. \begin{array}{l} v:m::a:b \\ v:n::b:a \end{array} \right\}$$

$\therefore v^2:mn::ab:ab$ , hence  $v^2 = mn$  and  $m:v::v:n$ . Now as a geometrical mean is greater than an arithmetical, half  $m + n$  is greater than  $v$ , consequently half the sum of the two weights on a false balance gives more than the true weight.

It follows also from this prop. that when two men carry a weight upon a lever lying on their shoulders, that the part which each bears is inversely as his distance from the weight.

A lever is defined to be an inflexible rod void of gravity and moveable about a fulcrum. But as every body has gravity, the lever is balanced



lanced before the weights are applied, so that no effect but that of the weights are to be considered. A straight lever is that where the points to which the weights are applied and the fulcrum are in the same straight line, in which case, if the lever be in equilibrio in one position, it will in all others, and therefore its weight may in all cases be neglected. A bent lever must also be so constructed as to be in equilibrio in any position, otherwise its property cannot be experimentally proved; that is, by constructing the lever so that the center of suspension may be the center of gravity. A straight lever having the forces applied obliquely becomes familiar to a bent lever, so far as the forces produce an equilibrium; for after resolving each force into two, one in the direction of the arm and the other perpendicular to it, the effect of the latter upon the arms to balance each other will be the same, whether the arms lie in the same direction from the fulcrum, or form there an angle.

Hence it also appears, that the effect of a weight upon a lever to turn it about, is as the weight multiplied into its distance from the fulcrum; for the effects are always equal when these products are equal, and therefore the effects must be measured by the products.

Two equal weights hanging at unequal distances from the fulcrum will balance, if the velocities with which they move, when put in motion, be equal. This is called the MECHANICAL PARADOX.

23. If the weights act obliquely on the arms of a bent lever, they will be inversely as the perpendiculars from the fulcrum to the lines of direction.

When the fulcrum is between the power and weight, the lever is called of the FIRST kind; if the weight be between the fulcrum and power it is said to be of the SECOND kind; and if the power be between the weight and fulcrum it is called of the THIRD kind.

24. In a compound lever, the power : the weight  
 $::$  product of the lengths of the arms of the levers  
 lying on the contrary sides of the fulcrums to the  
 power : product of the lengths of the other arms.

25. In the wheel and axle, the power : the weight  
 $::$  radius of the axle : radius of the wheel.

Cor. Hence if there be several wheels so constructed, that the periphery of one may act upon the axle of the other, the power : the weight  $::$  the product of the radii of all the axles : the product of the radii of all the wheels.

26. On the inclined plane, the power : the  
 weight

weight :: the sine of the plane's inclination : the cosine of the angle which the string going from the weight makes with the plane; and the weight : the pressure :: the cosine of the same angle : the sine of the angle between the directions of the power and weight.

Cor. 1. Hence the power to sustain a given weight on a given plane is least, when the string from the weight is parallel to the plane, and their ratio is then that of the height : the length of the plane; and the weight : the pressure :: the length : the base.

Cor. 2. When the string from the weight is parallel to the height, the power is equal to the weight, and the pressure vanishes.

Cor. 3. When the string is parallel to the base, the power, weight and pressure are as the height, base and length.

The pressure of the weight is here greater than the weight itself, because the power presses the weight upon the plane.

27. In a fixed pulley, the power is equal to the weight.

Although there is here no mechanical advantage, there is frequently a conveniency, by altering the direction in which the power is applied.

28. If the same string go round two sets of pulleys in two blocks, the power : the weight :: unity : the number of strings at the lower block.

In this case the lower block cooperates with the weight, by being raised with it.

29. If each pulley have a separate string fixed to something immoveable above, the power : the weight :: unity : that power of 2 whose index is the number of moveable pulleys.

Here all the moveable pulleys cooperate with the weight, as they rise with it.

30. If each pulley have a separate string fixed to the weight, the power: the weight ::  $1 : 2^n - 1$ , where  $n$  is the whole number of the pulleys.

In this set of pulleys, the moveable pulleys cooperate with the power by descending with it.

Hence in all the cases of the pulleys, the pulleys themselves must first be balanced, and then the proportion of the power to the weight will be as given in the propositions.

31. In the screw, the power : the weight :: the distance of two contiguous threads in a direction parallel to the axis of the screw : the circumference described by the power.

In the common screws the friction is generally equal to the power at least, for when the power is not applied the weight does not make the screw run down.

In all the above propositions respecting the equilibrium of the power and weight, there has been supposed to be an equilibrium between the parts of the machine before the power and weight were applied, and no friction to hinder either of them from giving motion to it, so that when both are applied, the least power added to either will destroy the equilibrium. In cases therefore where the friction is considerable, and its effect cannot be estimated, no experimental proof of the proposition can be applied. In machines made of wood, the friction is generally estimated at one third of the whole weight, that is, that if a power equal to one third of the weight applied to the machine could be applied to the weight without adding any more friction, it would overcome the friction; but as, by adding this power, more friction is added to the machine, therefore one third of that power applied must be again added to overcome the friction occasioned by that power, and so on ad infinitum; hence if  $W$  = the whole weight on the machine,  $\frac{1}{3}W + \frac{1}{9}W$

+ &c. ad inf. =  $\frac{1}{2}W$  the power to be applied to overcome the resist-

ance. Or if instead of the friction being  $\frac{1}{3}W$  if we suppose it  $\frac{1}{n}W$ ,

then  $\frac{1}{n-1}W$  is the power to be applied to overcome it.

32. In a machine compounded of any number of mechanic powers, the power is to the weight as the sum of the ratios expressing the ratio of the power to the weight in each.

For example, if a power and weight act upon the arms of a lever, and the weight lie upon an inclined plane to which the string is parallel, then the power is to the weight in a ratio compounded of the inverse



verse ratio of the perpendiculars from the fulcrum upon the lines of direction, and of the length of the plane to the height.

In every machine, by diminishing the power we increase the time, and in the same proportion, because the velocity of the power : the velocity of the weight (the velocities being estimated in the directions in which they oppose each other) :: the weight : the power. In the theory of this science we suppose all planes and bodies perfectly smooth, levers to have no weight, chords to be perfectly pliable, and machines to have no friction. The allowances to be made for the difference between theory and practice from these circumstances must be determined by experiment.

### ON THE CENTER OF GRAVITY.

Def. The center of gravity is that point in a body or system of bodies, by which, if it were suspended, it would rest in any position.

33. If a straight rod be balanced upon a fulcrum, and weights be hung upon each side, there will be an equilibrium when the sums of the products of each weight multiplied into its distance from the fulcrum on each side are equal.

This appears from observation the 5th. to prop. 22.

34. If any point be assumed in the rod, the distance of the center of gravity from that point will be equal to the sum of the products of each body multiplied into its distance from that point, (those products being reckoned negative when the bodies lie on the contrary side of that point to the center of gravity,) divided by the sum of the bodies.

35. The effect of any number of weights applied to a lever to turn it about any point, is just the same as if all the weights were collected into their common center of gravity.

For by the last observation it appears, that the sum of the products of each body multiplied into its distance from any point, is equal to the sum of all the bodies multiplied into the distance of their center of gravity from that point, and the effect to turn a lever about any point is measured by the body into its distance from that point; therefore con-

sidering all the bodies as placed in their center of gravity the effect must continue the same.

### 36. Any three bodies connected together have a center of gravity.

For the effect of any two is the same as if they were placed in their center of gravity, by the last; hence, conceive them to be placed there, and the center of gravity of that and the third body, is the center of gravity of the three.

Hence therefore any system of bodies, however situated, has a center of gravity.

It follows also from the same principle that every body has a center of gravity; for every body may be conceived to be resolved into an infinite number of corpuscles, any number of which, however situated, have a center of gravity.

Hence as the effect of the whole body is the same as if all the matter were collected into the center of gravity, we may conceive it to be all concentrated into that point, and the effect to produce an equilibrium, or to generate motion on a lever, will remain the same. Hence the PLACE of a body is understood to be that point where the center of gravity is.

But although the effect of a body to produce motion may be the same as when all the matter is conceived to be concentrated in its center of gravity, yet the effect produced will not be the same, owing to the inertia being different. In the doctrine of equilibrium we have only weight to consider, whereas in the doctrine of motion we have to consider both the power which gives motion and the resistance of the body from its inertia to oppose the communication of that motion.

### 37. If a body be suspended by a string and drawn from its perpendicular position, it will be accelerated by a force which is as the sine of its angular distance from the lowest point.

Hence the place of a body being denoted by its center of gravity, it follows that when any body is suspended, it cannot rest till its center of gravity comes to the lowest point, because in any other situation it will be acted upon by an accelerating force. Hence if a body, or system of bodies, be not suspended by a string, but by a fulcrum or axis passing through some point in it which is not the center of gravity, it will rest when the center of gravity is either directly above or below the point of suspension.

It is upon this principle that a double cone appears to roll up two inclined planes forming an angle with each other and lying in the same plane; for as it rolls up it sinks between them, and by that means the center of gravity actually keeps descending. To effect this the height of the planes must be LESS than the radius of the base of the cone:

cone: if the height be EQUAL to the radius, the body will rest in any part of it; and if the height be GREATER than the radius, it will descend. A cylinder may also roll up an inclined plane for a small distance, if it be loaded on one side with something heavier than itself, and that side be laid towards the top of the plane, for then the center of gravity being out of the axis towards that part, it will descend whilst the body rolls upwards.

Upon the same principle a body which would fall off a table, will not fall off although you hang a weight upon the part which does not rest upon the table, provided you, by that means, throw the center of gravity of the whole under the table.

Hence also you may easily balance a body resting upon a point on its under side, by hanging on a body at each end; for by that means you throw the center of gravity below the point of suspension, and then it brings itself to its lowest point, where it rests. Whereas before the bodies were hung on, the center of gravity was above the point of suspension, and unless it had been exactly over it, which it is almost impossible to accomplish, it would descend, and the body must fall.

Hence we have a very easy practical method of finding the center of gravity of any irregular plane figure. Suspend it by any point with the plane perpendicular to the horizon, and from the point of suspension hang a body suspended by a string, and draw a line upon the body where the string passes over; do the same for any other point of suspension, and where the two lines meet is the center of gravity. For the center of gravity being in each line, it must be at the point where they intersect.

The direction of the line by which a body at rest is suspended is the direction in which gravity acts, and if a line be so drawn from the center of gravity of a body, it is commonly called the LINE OF DIRECTION.

38. If a line be drawn from any angle of a triangle to bisect the opposite side, the center of gravity of the triangle will be upon that line and at the distance of two-thirds of it from the angle.

Hence we may find the center of gravity of any rectilinear figure: divide it into triangles and find the center of gravity each, and in each center of gravity conceive bodies to be placed equal in weight to its respective triangle, or weights proportional to the respective areas, and then find the center of gravity of all the bodies.

39. The center of gravity of a parabola lies in its axis at the distance of three-fourths of it from the vertex.

The center of gravity of bodies in general can be found only by a fluxional



fluxional calculation. In all regular bodies it is manifestly in the point which we commonly call the middle. In a cone it is in the axis at the distance of three-fourths of it from the vertex. In an irregular body it may happen that the center of gravity may not fall within the body.

40. If there be any number of bodies and perpendiculars be drawn from them to any plane, the distance of the center of gravity of all the bodies from that plane, is equal to the sum of the products of each body multiplied into its perpendicular distance from the plane divided by the sum of the bodies.

Hence by assuming three planes and finding the distance of the center of gravity from each, you determine the center of gravity of the bodies.

Hence also the sum of the motions of any number of bodies reduced to the same direction, will be equal to the motion of all the bodies placed in their center of gravity in the same direction.

41. If a circle be described about the center of gravity of any number of bodies reduced to that circle by lines drawn perpendicular to it, then the sum of the products of each body into the square of its distance from any point of the periphery is the same.

42. If two bodies move uniformly in two straight lines, their center of gravity will either be at rest or move uniformly in a straight line.

43. If a body equal to the sum of any two bodies be placed in their center of gravity, and the same quantities of motion in the same directions be communicated to it which are communicated to the two bodies, this body will move in the same line which the center of gravity of the two bodies describes, and with the same velocity.

Cor. 1. Hence equal and contrary motions communicated to any system

system of bodies will have no effect upon their center of gravity, for they would not disturb a body equal to the sum of them all placed in their center of gravity. For the same reason the motion of the center of gravity of any number of bodies will not be disturbed by their collision.

Cor. 2. Hence also the center of gravity of a system of bodies will not be disturbed by their mutual attractions, as the motions thus communicated are always equal and opposite. Hence the center of gravity of our system of planets is either at rest or moves on uniformly in a straight line. The latter is supposed by Dr. HERSCHEL to be the case, from the change which has been observed in the relative situation of some of the fixed stars.

44. If one body be at rest and another describe any curve, the center of gravity will describe a similar curve.

45. If a body be placed upon an horizontal plane, and the line of direction pass within the base, it will stand; if it pass without the base, it will fall.

This is manifest from the 1st. observation to prop. 37. for conceiving the base to be the fulcrum, the center of gravity is directly over it in one case but not in the other.

Our own motions are subject to this rule, which we observe without thinking of it. When a man stands upright, his center of gravity falls between his feet and he is supported; but if he lean forward, he throws the line of direction without his base, and he would fall if he did not put forward one of his feet so as to cause it to fall within. For this reason, a porter with a load on his back leans forward that the load may not throw the line of direction out of his base behind.

Upon this principle also it is that a body just hung upon the edge of a table will not fall off, because part of the body hanging under the table, the center of gravity of the whole is supported.

46. If a body be set upon an inclined plane, the line of direction will fall oblique to the base; in this case, if the line of direction fall within the base the body will slide down the plane without tumbling; but if it fall without the base, the body will roll, or partly slide and partly roll, according as the quantity of friction is greater or less. If there

there were no friction, the body, of whatever form, would slide without rolling.

Hence a globe on a perfectly smooth inclined plane would slide without rolling; for the force of gravity can give the body no rotatory motion, and as there is no friction there is no force which acts out of the center of gravity to give it a rotation.

#### ON THE COLLISION OF BODIES.

47. If two nonelastic bodies  $A$  and  $B$  move in the same straight line with velocities  $x$  and  $y$ , and strike each other, their common velocity after impact will be  $\frac{Ax \pm By}{A + B}$ , where the sign  $+$  must be taken when the bodies move in the same direction, and  $-$  when in opposite directions.

Cor. If  $A$  strike  $B$  at rest, the velocity after impact  $= \frac{Ax}{A + B}$ .

This, and the propositions respecting elastic bodies, depend upon the third law of motion, that the quantity of motion in the same direction is not altered by the action of two bodies on each other. When two nonelastic bodies meet, they act upon each other till they have acquired a common velocity, and then they move on together. Moreover, when two bodies act thus directly upon each other, their inertia must be simply as their velocities and quantities of matter, for the endeavour of each body to oppose the communication of motion must be simply as its respective motion, because these motions act at no mechanical advantage or disadvantage.

48. If  $A$  and  $B$  be two perfectly elastic bodies moving in the same direction with the velocities  $x$  and  $y$ , and  $A$  strike  $B$ , then  $A$ 's velocity after impact will be  $\frac{Ax - Bx + 2By}{A + B}$ , and  $B$ 's will be  $\frac{2Ax - Ay + By}{A + B}$ . If they move in opposite directions,



tions,  $A$ 's velocity will be  $\frac{Ax - Bx - 2By}{A + B}$ , and

$B$ 's will be  $\frac{2Ax + Ay - By}{A + B}$ .

When two perfectly elastic bodies meet, they first act upon each other till they have acquired a common velocity, as in nonelastic bodies, and then by the endeavour of each body to recover its figure they separate; and when the force with which they separate is equal to the force with which they were compressed together, they are said to be perfectly elastic. The time in which the action between the bodies takes place is of no consequence; also the effect will be the same if only one body be elastic, for in that case they will first act upon each other till they have acquired a common velocity, as in nonelastic bodies, and then the force with which the elastic body endeavours to restore its figure will double the action between them, and therefore if the effect of the compression be the same, the effect of their separation must also be the same, and consequently the whole effect will be the same. If the force with which they separate be less than the force with which they are compressed together, they are said to be imperfectly elastic.

Cor. 1. If  $A$  strike  $B$  at rest, or  $y = 0$ ,  $A$ 's velocity after impact  $= \frac{Ax - Bx}{A + B}$ , and  $B$ 's  $= \frac{2Ax}{A + B}$ . Hence if  $A = B$ ,  $A$  will be at rest after impact, and  $B$  will move with the velocity which  $A$  had before impact. If  $B$  be greater than  $A$ ,  $A$ 's velocity becomes negative, and therefore  $A$  will be reflected back. If  $B$  be less than  $A$ ,  $A$ 's velocity will be positive, and therefore  $A$  will proceed on in the same direction after impact.

Hence if there be any number of bodies of equal magnitude lying in the same straight line, and the first strike the second, all the bodies will rest except the last, which will move off with the velocity of the first before impact. This theory will be found to agree very nearly with experiments made with ivory balls, which are nearly perfectly elastic. If the bodies increase in magnitude, each will be reflected back, and if they decrease, each will go forward after the stroke.

Cor. 2. If  $A = B$  and they move in the same direction,  $A$ 's velocity after impact  $= y$ , and  $B$ 's velocity  $= x$ , hence the bodies have exchanged velocities and continue to move in the same direction. If they move in opposite directions each will be reflected back, having exchanged velocities.

Cor. 3. If  $A$  strike an immoveable object, or if  $B$  be infinite and  $y = 0$ ,  $A$ 's velocity after impact  $= -x$ , or  $A$  will be reflected back with the same velocity.

Cor. 4. The velocity of  $A$  minus that of  $B$  before impact  $=$  the velocity of  $B$  minus that of  $A$  after; for  $B$ 's velocity  $- A$ 's after impact, when they move in the same direction, is  $\frac{2Ax - Ay + Bx - Ax + Bx - 2By}{A + B}$

$$= \frac{Ax - Ay + Bx - By}{A + B} = \frac{A + B \times x - y}{A + B} = x - y. \text{ If the bodies move}$$

in the opposite directions,  $B$ 's velocity  $-A$ 's  $= x + y$ , which is the difference of the velocities, estimated in the same direction, before impact.

Cor. 5. If  $p$  and  $q$  be the velocities of  $A$  and  $B$  after impact,  $Ax^2 + By^2 = Ap^2 + Bq^2$ . For if the bodies move in the same direction, by the third law of motion  $Ax + By = Ap + Bq$ , also  $x - y = q - p$ , hence

$$\left. \begin{array}{l} A \times x - p = B \times q - y \\ x + p = q + y \end{array} \right\}, \text{ hence } Ax^2 + By^2 = Ap^2 + Bq^2. \text{ If the}$$

$\therefore A \times x^2 - p^2 = B \times q^2 - y^2$   
bodies move in opposite directions, the same conclusion follows from the same principles.

49. If the bodies be imperfectly elastic, and  $m : n ::$  the force with which they are compressed together till they acquire a common velocity, or perfect elasticity, : the force with which they separate, or their imperfect elasticity; then if they move in the same direction,  $A$ 's velocity after impact will

$$\text{be } x - \frac{m+n}{2m} \times \frac{2Bx - 2By}{A+B}, \text{ and } B\text{'s will be } y +$$

$$\frac{m+n}{2m} \times \frac{2Ax - 2Ay}{A+B}. \text{ If they move in opposite}$$

directions, the signs of the terms where  $y$  enters must be changed.

Hence if we have the magnitudes of the bodies  $A$  and  $B$ , and their velocities  $x$  and  $y$  before impact, and their velocities after, we shall have two equations from which we can determine the ratio of  $m : n$ , or the degree of elasticity of the bodies. The velocities after may be found by letting the bodies vibrate through equal arcs and meet at the lowest point, and then measuring the arc each describes after impact. Since the velocity is as the chord of the arc, if we measure only the chords in their descent and ascent it will give their relative velocities, which will be sufficient. If  $A$  strike  $B$  at rest and be equal to it,  $B$ 's

$$\text{velocity after} = \frac{m+n}{2m} \times x; \text{ hence } A\text{'s velocity before} : B\text{'s after} :: x :$$

$$\frac{m+n}{2m} \times x :: 2m : m+n :: \text{the chord } a \text{ described by } A : \text{the chord } b \text{ described by } B; \text{ hence } m : n :: a : 2b - a.$$

ON THE CENTERS OF PERCUSSION, SPONTANEOUS  
ROTATION, OSCILLATION AND GYRATION OF  
BODIES.

Def. 1. The center of *percussion* is that point in a body revolving about an axis, at which, if it struck an immoveable obstacle, all its motion would be destroyed, or it would incline neither way.

From this def. it appears, that the point of suspension is not affected by the stroke.

2. The center of *spontaneous rotation* is that point which remains at rest the instant a body is struck, or about which the body begins to revolve.

Hence the center of spontaneous rotation is the same as the center of suspension corresponding to the center of percussion, the center of percussion being the point where the body is struck. For the action of the body against the immoveable obstacle in the center of percussion must have the same effect upon the body as if the body had been at rest and the obstacle had struck the body, in which latter case the center of suspension would not be affected, and therefore it becomes the center of spontaneous rotation.

3. The center of *oscillation* is that point in a vibrating body at which, if a corpuscle were suspended, it would vibrate in the same time the body does.

4. The center of *gyration* is that point in a body, into which, if the whole quantity of matter were collected, the same moving force would generate in it the same angular velocity.

50. If a body revolve about an axis, the effect of any particle  $p$  to resist, by its inertia, the communication of motion to any point, is as the particle multiplied into the square of its distance from the axis.

For the inertia of any particle not acting at any mechanical advantage or disadvantage to oppose the communication of motion, is as its velocity multiplied into its quantity of matter  $p$ , or in this case as  $d \times p$ , if  $d$  be the distance of the particle from the axis, the velocity of each particle varying as  $d$ . But this inertia acting upon a lever whose length is  $d$  in opposition to a force acting at any other point, the effect of the inertia, by the property of the lever, will be  $d^2 \times p$ .

51. If any number of bodies  $A, B, C$ , revolve about an axis at the respective distances  $a, b, c$ , and  $x$  be the distance of the center of gyration



from the axis, then  $x = \sqrt{\frac{a^2 \times A + b^2 \times B + c^2 \times C}{A + B + C}}$ .

For the inertia of  $A + B + C$  placed at the distance  $x$  from the axis is  $x^2 \times A + B + C$ ; now as the moving force is the same, the same angular velocity will be generated when the inertia is the same; hence  $x^2 \times A + B + C = a^2 \times A + b^2 \times B + c^2 \times C$ , therefore  $x = \sqrt{\frac{a^2 \times A + b^2 \times B + c^2 \times C}{A + B + C}}$ . If the axis pass through the center of gravity it is called the *principal* center of gyration.

If a slender rod whose length  $= a$  revolve about one end, the distance of the center of gyration from that end  $= a \sqrt{\frac{1}{3}}$ . Or if it re-

volved about its center, and  $a$  were equal to half its length, the distance of the center of gyration from the center would be the same. If a circle revolve in its own plane about its center, or a cylinder about its axis, and  $r$   $=$  the radius, the distance of the center of gyration from

the center or axis  $= r \sqrt{\frac{1}{2}}$ . If a globe whose radius is  $r$  revolve about one of its diameters, the distance of the center of gyration from the center  $= r \sqrt{\frac{2}{5}}$ .

Cor. 1. If a circle be described about the center of gravity, and any point of its periphery be made the axis of rotation, the distance from it to the center of gyration will remain the same, the plane of rotation continuing the same, This appears from prop. 41.

Cor. 2. To find what quantity of matter  $\mathcal{Q}$  must be placed at any other distance  $d$  from the axis so that the inertia may remain the same, we have  $d^2 \times \mathcal{Q} = x^2 \times A + B + C$ , hence  $\mathcal{Q} = \frac{x^2}{d^2} \times A + B + C$ .

52. If a body vibrate about an axis by the force of gravity, the distance of the center of oscillation from the axis is equal to the sum of the products of each particle multiplied into the square of its distance from the axis, divided by the body multiplied into the distance of the center of gravity from the axis. The center of percussion is the same as the center of oscillation.

Let

Let  $LM$  (fig. 1.) be a plane passing through the center of gravity  $G$  of the body, perpendicular to the axis of vibration, on which the body is orthographically projected;  $O$  the center of oscillation in the line  $CG$  produced;  $a, b, \&c.$  the constituent particles of the body thus projected,  $Cm$  parallel to the horizon, and draw  $Om, Gg, aq, bp, \&c.$  perpendicular to  $Cm$ . Now as the angular velocity of each particle of the body is not altered by this projection, we have by  $a \times aC^2 + b \times bC^2 + \&c. =$  the inertia of the whole body; also if  $O$  represent a particle at  $O$ ,  $O \times OC^2 =$  the inertia of that particle. Now it appears from prop. 23. that obf. 5. prop. 22. is true in general, if you assume the perpendicular distances from the fulcrum instead of the real distances; therefore  $a \times qC + b \times pC + \&c. =$  the effect of gravity to turn the whole body about  $C$ , and  $O \times mC =$  the effect of gravity to turn the particle  $O$  about  $C$ . Hence, that the same angular velocity may be generated in both cases, the accelerative forces must be in proportion to the respective inertias; that is  $a \times qC + b \times pC + \&c. : O \times mC ::$

$$a \times aC^2 + b \times bC^2 + \&c. : O \times OC^2, \text{ therefore } OC^2 = \frac{a \times aC^2 + b \times bC^2 + \&c.}{a \times qC + b \times pC + \&c. \times mC} = \frac{a \times aC^2 + b \times bC^2 + \&c. \times mC}{a + b + \&c. \times Cg}; \text{ but } CO : mC :: CG : Cg, \therefore \frac{CO}{CG} = \frac{mC}{Cg}; \text{ hence } OC = \frac{a \times aC^2 + b \times bC^2 + \&c.}{a + b + \&c. \times CG}.$$

The center of oscillation thus found being independent of the line  $Cm$ , shows that it is a fixed point for every position of the body. Hence if a body could have all its matter concentrated in  $O$  and be suspended at  $C$ , it would perform all its vibrations in the same time the body does. Hence any body  $LM$  thus vibrating, may be considered as a pendulum whose length is  $CO$ , so far as regards the time of vibration.

The center of oscillation of a rod, vibrating about one end, is two thirds of its length from that end. If a sphere be suspended at the distance of  $d$  from its center, and  $r$  be its radius, the distance from the center of suspension to the center of oscillation  $= d + \frac{2r^2}{5d}$ . Hence if

that distance  $a$  be given, and also  $r$ , we have  $d = \frac{a}{2} \pm \sqrt{\frac{a^2}{4} - \frac{2r^2}{5}}$ ,

which shows that there are two points by which it may be suspended

to vibrate in the same time. When  $\frac{a^2}{4} = \frac{2r^2}{5}$ , or  $a = r \sqrt{\frac{8}{5}}$ , there is

only one value of  $d$  which  $= \frac{a}{2} = r \sqrt{\frac{2}{5}}$ , which gives the point of

suspension the center of gyration, in which case  $a$  is the least possible, for if  $a$  be assumed less,  $d$  becomes impossible. This is agreeable to what is shown in a subsequent note to this prop. Hence we know  $d - r$  the distance from the surface at which it must be suspended to make the length of the pendulum  $= a$ , which is more convenient than measuring

suring from the center. If  $a$  be not very small in comparison with  $r$ , we may take  $d = a$  without any sensible error. The diameter of the ball made use of in our experiments is 1,6 in. therefore the distance of the point of suspension from the surface  $= a - 0,8$  in.

If a rod in the form  $\perp$  whose base  $= 20$  in. and perpendicular  $= 12$ , the perpendicular bisecting the base, be suspended at the upper end and vibrate in its own plane, the distance from the point of suspension to the center of oscillation  $= 13,21$  inches; but if it vibrate perpendicular to its plane, the distance of the centers  $= 11,08$  inches. If the rod be suspended at the distance of 6 inches from the top, the distance of the point of suspension from the center of oscillation will be 12,75 inches in the former case, and 7,2 inches in the latter. Hence by the last observation the distance of the points of suspension from the surface of the sphere which shall vibrate in the same times are 12,41; 10,28; 11,95 and 6,4 inches respectively.

The distance of the center of gyration from the axis of motion, is a mean proportional between the distances of the centers of oscillation and gravity from the same axis.

Hence the time of vibration will be the least possible when the axis passes through the principal center of gyration. For if  $x$  and  $a$   $=$  the distances from the center of gravity to the centers of suspension and gyration, then  $x : x + a :: x + a : \frac{x + a^2}{x}$  the length of the pendulum, which

is the least when  $x = a$ .

The product of the distances of the center of gravity from the axis of suspension and center of oscillation is a constant quantity for the same plane of vibration; if therefore the center of oscillation be made the point of suspension, the point of suspension becomes the center of oscillation. Hence also it follows, that if upon the plane of vibration passing through the center of gravity of any body, two circles be described with the center of gravity as their center and radii equal to the distances of the center of gravity from the point of suspension and center of oscillation, the body suspended from any point in the periphery of either circle, will perform its vibration in the same time. Hence there are, an indefinite number of points in the same body, by which, if the body were suspended, the time of vibration would remain the same.

Hence also if the point of suspension and center of oscillation be given in one case, and also any other point of suspension, the center of oscillation will be known, and consequently the length of the pendulum, the plane of vibration remaining the same. For example, if a slender rod 36 inches long be suspended at one end, the distance of the center of oscillation will be 24 inches from it, and their distances from the center of gravity 18 and 6 inches respectively; hence if the same rod be suspended 10,39 inches from the center of gravity, then will  $\frac{18 \times 6}{10,39} = 10,39$  inches, be the distance from the center of gravity to the center of oscillation; hence the length of the pendulum  $= 20,78$  inches. Here the principle center of gyration is the point of



of suspension, consequently the time of vibration is the least possible. If the rod be suspended 5,2 inches from the center of gravity, the length of the pendulum = 26,17 inches. If it be suspended 3,25 in. from the center of gravity, the length of the pendulum = 36,48 in.

If the same body vibrate about an axis at the same distance from the center of gravity but in a different position, the time of vibration will not be the same, unless the sum of the products of each particle  $\times$  the square of its distance from the axis remains the same.

To find the center of oscillation of an irregular body suspended at any point, hang up a simple pendulum, that is a small globe suspended by a string, and adjust its length till it vibrates in the same time the body does, and the length of that pendulum is equal to the distance of the point of suspension from the center of oscillation of the body. Or two thirds of the length of a slender rod suspended at one end and vibrating in the same time, will give the same.

If a body, instead of revolving about a center be made to move parallel to itself by being conceived to be suspended at an infinite distance, the centers of gravity, oscillation and percussion become the same.

53. If a body vibrate, and every particle be attracted to a center by a force which varies as its distance from it, the center of oscillation will be the same as when it is acted upon by a constant force acting in parallel lines. If the force vary in any other ratio, the center of oscillation will not be the same, nor will it continue a fixed point for a whole vibration, but will vary as the position of the body varies.

54. If two bodies  $p$  and  $q$  hang upon a lever at the distances  $m$  and  $n$ , and  $p$  descend, the pressure

$$\text{upon the axis} = \frac{m p - n q \times p q}{m^2 p + n^2 q}.$$

Let  $a$  and  $c$  be the distances of the centers of oscillation and gravity from the center of suspension; then as the whole system revolves with the same angular velocity as if the whole quantity of matter  $p + q$  were placed at the distance  $a$  from the fulcrum, and as the accelerative forces are as the velocities generated in a given time, or as the distances from

the fulcrum, we have  $a : c :: p + q : \frac{c \times p + q}{a}$  the force with which the

center

center of gravity descends, and which  $= \frac{m^2 p - n^2 q}{m^2 p + n^2 q}$  because  $a = \frac{m^2 p + n^2 q}{m p - n q}$  and  $c = \frac{m p - n q}{p + q}$ ; but the force with which the center of gravity descends is that by which the pressure upon the axis is diminished; hence the pressure upon the axis  $= p + q - \frac{m^2 p - n^2 q}{m^2 p + n^2 q} = \frac{m + n}{m^2 p + n^2 q} \times p q$ .

Cor. If  $m = n$ , the pressure  $= \frac{4 p q}{p + q}$ . If  $p = 4$  oz.  $q = 3$  oz. the pressure  $= 6$  oz. 17 drs. 3 grs.

#### ON THE MOTION OF BODIES ACTED UPON BY UNIFORMLY ACCELERATING FORCES.

Def. A force is said to be uniformly accelerating, when the quantity of acceleration continues to be the same in the same time, or when equal increments of velocity are generated in equal times.

55. If a body move uniformly with the velocity  $V$  for the time  $T$ , the space described  $= TV$ .

In mechanics we measure the velocity by the space which a body describes in one second, supposing the velocity to be continued uniform for that time; also the time is estimated in seconds, and the space in feet. Hence, as the spaces must be in proportion to the times when the velocity is uniform,  $1'' : T'' :: V : TV$  the space described. It may perhaps appear to be improper to multiply time and velocity, two heterogeneous quantities, together; but it must be observed, that  $T$ , in the expression  $TV$ , is an abstract number, being the quotient of  $T''$  divided by  $1''$ ; the abstract number therefore by which  $V$  is multiplied is always the same as that which denotes the number of seconds in the given time.

A quantity is said to be given when it continues the same whilst the other quantities with which it is connected vary. Thus, if the time remain the same whilst the velocity varies, or if two bodies move with different uniform velocities for the same time, the time is said to be given, and the spaces will be in proportion to the velocities.

56. The space described in the last proposition may be represented by the area of a right angled parallelogram, one of whose sides represents the time and the other the velocity..

For

For its area is equal to the product of one side  $\times$  into the other.

57. If the force of gravity be denoted by unity, and  $F$  be any other uniformly accelerative force compared with it, also if  $v$  be the velocity generated by gravity in one second, and  $V$  be the velocity generated by the force  $F$  in the time  $T$ , then  $V = FVv$ .

For by the same force the velocity generated must be as the time, equal velocities being generated in equal times, hence  $1'' : T'' :: v : T v$  the velocity generated by gravity in the time  $T$ . Also for the same time the velocity generated must evidently be as the force, hence  $1$  (grav.) :  $F :: T v : V = F T v$ . Here  $F$  and  $T$  become abstract numbers, being equal to the quotients of force divided by force, and time by time. Hence as  $v$  is given,  $V$  varies as  $F T$ .

58. The accelerative force of a body varies as the moving force directly and the quantity of matter to be moved inversely.

For the moving force is in proportion to the quantity of motion generated by it in a given time, or as the velocity  $\times$  the quantity of matter; therefore the moving force divided by the quantity of matter varies as the velocity, which varies as the accelerative force, when the time is given, by the last prop.

All bodies descend with equal velocities by the force of gravity, and therefore the moving force must be in proportion to the quantity of matter, for to make twice the quantity of matter descend with the same velocity, twice the force must manifestly be applied. We may therefore here estimate the moving force by the quantity of matter to be moved, and make the accelerative force of gravity unity; and then any other accelerative force may be compared with it, by measuring the force of a body by its quantity of matter when it hangs freely down, or by its quantity of matter, when it moves in any other direction, multiplied into the force of gravity in that direction, because the body must have a less force in any other direction as gravity in that direction is less. If two bodies  $P$  and  $Q$ ,  $P$  being the greatest, hang over a pulley, the moving force is  $P - Q$ , that being the quantity which gives motion to the bodies; but this moving force has both bodies to move, and therefore must move them with less velocity, that is, less accelerate them, or the accelerative force will be less, the greater the bodies are, or the accelerative force will, from this cause only, be inversely as  $P + Q$ . But it is manifest that a greater moving force must, *ceteris paribus*, give a proportionably greater acceleration, Hence the



whole accelerative force  $= \frac{P-Q}{P+Q}$ . In this case the force of gravity  $= 1$ ; for if  $Q = 0$ ,  $P$  falls freely by gravity, and the accelerative force becomes unity.

59. If a body fall from a state of rest and be acted upon by any uniformly accelerative force  $F$ , and  $T$  be the time of its acting,  $V$  the velocity acquired at the end of that time, and  $S$  the space, then  $S$  varies as  $T \times V$ , or as  $F \times T^2$ ; also  $V^2$  varies as  $F \times S$ .

Here  $F$  and  $T$  are to be considered as abstract numbers, as explained in prop. 57.

Cor. 1. If  $F$  be given, that is, if bodies fall and be acted upon by the same uniformly accelerating force, then  $S$  varies as  $T^2$ , and also as  $V^2$ . Hence if we take a succession of times as 1, 2, 3, 4, 5, 6, &c. the spaces will be as 1, 4, 9, 16, 25, 36, &c. consequently the spaces described in the equal successive portions of time will be as the odd numbers 1, 3, 5, 7, 9, &c.

Cor. 2. If the time be given, the space varies as the force, or as the velocity generated.

60. If a body fall from a state of rest and be acted upon by an uniformly accelerative force, and in descending through the space  $S$  in the time  $T$  it acquire a velocity  $V$ , the body with the last acquired velocity  $V$  continued uniform for the time  $T$  would pass over the space  $2S$ .

In our latitude, a heavy body is found by experiment to descend  $16\frac{1}{2}$  feet in the first second, hence the velocity acquired in that time is  $32\frac{1}{2}$  feet in a second. All bodies would fall with the same velocity, were it not for the resistance of the air, for in an exhausted receiver, a guinea and a feather fall from the top to the bottom in the same time. Hence the force of gravity acts equally upon every kind of matter,

61. If a body fall freely by the force of gravity through the space  $S$ , and  $T$  be the time of descent and  $V$  the velocity acquired, also  $m = 16\frac{1}{2}$  feet; then

then 1<sup>st</sup>.  $S = mT^2$ ; 2<sup>dly</sup>.  $T = \sqrt{\frac{S}{m}}$ ; 3<sup>dly</sup>.  $S = \frac{V^2}{4m}$ ;  
 4<sup>thly</sup>.  $V = \sqrt{4mS}$ ; 5<sup>thly</sup>.  $V = 2mT$ , and 6<sup>thly</sup>.  $T = \frac{V}{2m}$ .

62. If a body fall freely by any other force  $F$  compared with gravity represented by unity, then

$$S = mFT^2, V = \sqrt{4mFS} \text{ and } T = \frac{V}{2mF}.$$

63. If  $M$  represent the moving force, estimated as explained in prop. 58. and  $\mathcal{Q}$  the quantity of matter to be moved, then  $F = \frac{M}{\mathcal{Q}}$ ; hence, by the

$$\text{last prop. } S = \frac{M}{\mathcal{Q}} \times mT^2, V = \sqrt{\frac{4mMS}{\mathcal{Q}}}, \text{ and } T = \frac{V\mathcal{Q}}{2mM}.$$

Cor. 1. If two bodies  $P$  and  $\mathcal{Q}$  hang over a pulley, of which  $P$  is the greatest, then  $P - \mathcal{Q}$  is the moving force, and  $P + \mathcal{Q}$  the quantity of matter to be moved; hence  $S = \frac{P - \mathcal{Q}}{P + \mathcal{Q}} \times mT^2$ . But we have not here taken into consideration the quantity of matter in the pulley to be moved; now as the pulley has a rotatory motion, the different parts of which move with different velocities from that of  $P$  and  $\mathcal{Q}$ , except its circumference, we must not add the quantity of matter in the pulley to  $P + \mathcal{Q}$  in order to get the whole inertia, but we must compute by prop. 51. cor. 2. what quantity of matter  $q$  placed in the circumference will retard just as much as the whole pulley; and then

$$S = \frac{P - \mathcal{Q}}{P + \mathcal{Q} + q} \times mT^2; \text{ also } V = \sqrt{\frac{4m \times P - \mathcal{Q} \times S}{P + \mathcal{Q} + q}}, \text{ and } T = \frac{V \times P + \mathcal{Q} + q}{2m \times P - \mathcal{Q}}.$$

If the body be irregular,  $q$  cannot be computed, but may be thus determined by experiment. With two given weights  $P, Q$ , observe the space  $S$  described in any time  $T$ , and then  $q = \frac{P - Q \times m T^2}{S}$

$- P - Q$

To take away all resistance, as far as possible, the axis of the wheel over which  $P$  and  $Q$  hang, should lie upon friction wheels, by which the friction becomes insensible, and the experiments will answer to the theory without any sensible difference.

By a variety of experiments to determine the inertia of the friction wheels made use of in our experiments, it appears that  $q = 2,75$  oz. troy. The machine for these experiments was invented by Mr. ATWOOD, and is most admirably adapted to the purposes for which it was intended, as the whole theory of uniformly accelerating and retarding forces may be proved by it, as the author himself has shown in his *Theory of rectilinear and rotatory Motion of Bodies*, and in his excellent *Analysis of a Course of Lectures on the Principles of Natural Philosophy*, read at Cambridge. By altering the value of  $P, Q, S$  and  $T$ , and by supposing some given whilst the others vary, the truth of all the principles may be experimentally proved. And by taking off weight from  $P$  as it ascends till it becomes less than  $Q$ , the motion at that instant becomes retarded; and from thence every thing relating to uniformly retarding forces may be proved. If in the descent of  $P$ , so much weight be taken from it as to leave it equal to  $Q$ , the bodies go on without any acceleration, and it appears that they then describe twice the space which they had before described in the same time. GALILEO, who first gave the theory of uniformly accelerating forces, proved the general law, that the spaces vary as the squares of the times, by letting bodies descend upon inclined planes. But on account of the friction of the planes he could not apply the method to the absolute quantities of space, time and velocity, nor to the case where the moving force and the quantity of matter to be moved are different.

Cor 2. If  $Q$  lie on an horizontal plane, the moving force is  $P$  only, and hence  $S = \frac{P}{P + Q} \times m T^2$ , neglecting the inertia of the pulley over which the string runs.

Cor. 3. If  $P$  hang freely down, and  $Q$  lie upon an inclined plane with the string parallel to it, then if the height of the plane be to its length  $:: r : s$ , the force of  $Q$ 's descent  $= \frac{rQ}{s}$ , and if  $P$  draw  $Q$  up, the moving force  $= P - \frac{rQ}{s} = \frac{sP - rQ}{s}$ , hence  $S = \frac{sP - rQ}{s \times P + Q} \times m T^2$ , neglecting the inertia of the pulley.

64. If a body descend down an inclined plane, the accelerative force : the force of gravity  $::$  the height



height  $H$  : the length  $L$ , or, as radius : the cosine of the plane's inclination.

Cor. 1. Hence as the force of gravity is constant at the same place, the accelerative force varies as the height of the plane directly and length inversely, and therefore is uniform for the same plane.

Cor. 2. If we denote the force of gravity of a body by its weight  $W$ , its accelerative force upon the plane  $= \frac{H}{L} \times W$ .

65. If a body descend down an inclined plane without friction, the velocity acquired is the same as that down the height.

Hence the velocities down all planes of the same height are equal, and vary as the square roots of the heights.

66. If a body could descend without friction down several inclined planes connected together, and lose no motion in going from one to another, the velocity acquired would be the same as that down the height of the whole system.

67. When a body descends from one inclined plane to another, the velocity is diminished in the ratio of radius : the cosine of the angle between the directions of the planes.

68. If a body descend without friction down any curve, the velocity is the same as that which would be acquired down its altitude.

Hence if a body descend in any curve, it will ascend to the same altitude in whatever curve it may rise.

Hence when a body is suspended by a string and made to vibrate in any curve, the velocity is that which is acquired in falling down the altitude, there being no friction to retard it.

69. If a body descend down an inclined plane, the time varies as  $\frac{L}{\sqrt{H}}$ .

Cor:

Cor. Hence when  $H$  is given, or two inclined planes have the same height, the time varies as the length. Hence the time down the length : the time down the height ::  $L : H$ ; but by prop. 61. the time down  $H$  is equal to  $\sqrt{\frac{H}{m}}$ , hence the time down  $L = \frac{L}{H} \times \sqrt{\frac{H}{m}}$ .

70. If the diameter of a circle be perpendicular to the horizon, and chords be drawn from either extremity, the times of descent down all the chords are equal, and the velocities and accelerative forces will be as the lengths of the chords.

71. The times down similar systems of inclined planes, similarly situated, vary as the square roots of their lengths, supposing there be no friction, nor any motion lost at the angles.

Cor. Hence as a curve may be considered as the limit to which a rectilinear figure approaches by diminishing the length of its sides and inclination to each other *sine limite*, the times down similar curves vary as the square roots of their lengths.

Thus far we have considered the motion of bodies acted upon by constant accelerative forces; the next proposition contains the general principles of motion when the forces are variable.

72. If a body descend in any line by any constant or variable force  $F$  compared with gravity represented by unity,  $x$  = the space described,  $v$  the velocity,  $t$  the time, and  $m = 16\frac{1}{2}$  feet, then

$$v \dot{v} = \pm 2 m F \dot{x}; \text{ also } \dot{t} = \pm \frac{\dot{x}}{v}.$$

For  $\dot{v}$  varies as  $F \times \dot{x}$ ; but  $\dot{t}$  varies as  $\pm \frac{\dot{x}}{v}$ ; hence  $\dot{v}$  varies as  $\pm \frac{F \dot{x}}{v}$ ,  $\therefore v \dot{v}$  varies as  $\pm F \dot{x}$ . Now by prop. 61.  $v^2 = 4 m x$  by gravity, hence  $v \dot{v} = \pm 2 m \dot{x}$ ,  $\therefore v \dot{v} : \pm 1 \times \dot{x} :: 2 m : 1$ ; but  $v \dot{v}$  is in a constant ratio to  $\pm F \dot{x}$ ; hence if we consider 1 as the force of gravity, that ratio is  $2 m : 1$ ; hence  $v \dot{v} : \pm F \dot{x} :: 2 m : 1$ , consequently  $v \dot{v} = \pm 2 m F \dot{x}$ . The sign + must be used when  $v$  and  $x$  increase or decrease together, and - on the contrary.

Also as  $v$  denotes the space described uniformly in 1", and  $\dot{x}$  is described with the same velocity in the time  $\dot{t}$ ;  $\therefore$ , as the velocity is given,

$v :$

$v : 1'' :: \pm \dot{x} : t = \pm \frac{\dot{x}}{v}$ , where + must be used when  $t$  and  $x$  increase or decrease together, and — on the contrary.

73. If a body descend from rest in a right line towards a center of force, and  $x$  be the distance from that center; and if at the distance  $d$  the force  $= c$  compared with gravity represented by unity, then, if the force vary as the  $n^{\text{th}}$ . power of the distance,  $d^n : x^n :: c : \frac{c x^n}{d^n}$  the force at the distance

$$x; \text{ hence } v \dot{v} = - \frac{2 m c x^n \dot{x}}{d^n}, \therefore v^2 = \frac{- 4 m c}{n + 1 \times d^n} \times x^{n+1}$$

+ cor. but when  $x = a$ , the greatest distance,  $v = 0$ ,

$$\text{hence } v^2 = \frac{4 m c}{n + 1 \times d^n} \times a^{n+1} - x^{n+1}, \text{ therefore } v =$$

$$\sqrt{\frac{4 m c}{n + 1 \times d^n} \times \sqrt{a^{n+1} - x^{n+1}}}. \text{ Also } t = - \frac{\dot{x}}{v} =$$

$$\frac{- \dot{x}}{\sqrt{\frac{4 m c}{n + 1 \times d^n} \times \sqrt{a^{n+1} - x^{n+1}}}}, \text{ whose fluent,}$$

which cannot be expressed in general, properly corrected, gives  $t$ .

ON THE VIBRATION OF PENDULUMS IN THE ARCS OF CIRCLES AND CYCLOIDS, AND THEIR MOTIONS IN CONICAL SURFACES.

74. If a pendulum vibrate in the arc of a circle, it is accelerated by a force which is to the force of gravity,



gravity, as the sine of its angular distance from the lowest point to radius.

Cor. Hence in the same pendulum, the accelerative force is as the sine of its distance from the lowest point.

75. If a pendulum vibrate in a circular arc, the longer the vibration is the longer will be the time.

It will appear for prop. 81. and cor. prop. 84. that if the accelerating force varies as the distance from the lowest point, all the vibrations will be performed in the same time; but here the accelerating force varies in a less ratio, for the sine varies slower than the arc; hence by increasing the arc the force does not increase fast enough to make all the vibrations equal, and therefore the greater arcs having too small a force for that purpose, they will be described in greater times.

In very small arcs, where the sine and arc are very nearly equal, all the vibrations, as to sense, will be performed in the same time.

76. The times in which different pendulums perform vibrations in similar arcs of circles are as the square roots of their lengths.

For by cor. prop. 71. the times vary as the square roots of the arcs, and the arcs are as the radii.

As the length of the pendulum is denoted by the distance from the point of suspension to the center of oscillation, the arc described by the pendulum is always understood to be the arc described by its center of oscillation.

77. If a pendulum hang at rest and a body strike it, then having given the quantity of matter in the striking body and in the pendulum, also the point of impact of the pendulum, and the circular arc which it describes, the velocity of the striking body may be found.

Let  $x$  be the distance from the point of suspension to the center of gyration of the pendulum,  $d$  the distance from the point of suspension to the point of impact,  $m$  the quantity of matter in the pendulum,  $M$  the quantity of matter in the body,  $v$  the velocity communicated to the point struck,  $V$  the velocity of the body, then by cor. 2. prop.

51. if a quantity of matter  $= m \times \frac{x^2}{d^2}$  be placed at the point of impact,

the

the same angular velocity will be generated; hence by the rule for the collision of nonelastic bodies,  $M + m \times \frac{x^2}{d^2} : M :: V : v$ , hence  $V =$

$$\frac{v M d^2 + v m x^2}{M d^2}.$$

Now to determine  $v$ , observe what arc the center of oscillation of the pendulum describes in its ascent after impact, and find its versed-sine  $r$ , then the velocity of the center of oscillation at the lowest point  $= \sqrt{4 m r}$  by prop. 61. the velocity down the versed-sine being the same as that in the arc by prop. 68. Now let  $b$  be the distance from the point of suspension to the center of oscillation, then  $b :$

$$d :: \sqrt{4 m r} : v = \frac{d \sqrt{4 m r}}{b}.$$

In this manner Mr. ROBINS determined the initial velocity of balls fired from a gun. To the bottom of the pendulum a ribband was fixed which passed between two steel edges pressing against each other, so that the length of the ribband drawn out gave the chord of the arc described.

### 78. The force of gravity is exerted upon every body in proportion to its quantity of matter.

For bodies of different quantities of matter describe the same arc in the same time, and therefore the greater the quantity of matter the greater must be the force exerted upon it in the same proportion. Weight also being the effect of attraction as the cause, must be a relative quantity, the same body weighing differently on different parts of the earth according as the attraction varies.

LEM. If  $v$  = the velocity of a body revolving in a circle whose radius  $= r$ , and the force of gravity be represented by  $32\frac{1}{2}$  feet, the centrifugal force of the body in the circle  $= \frac{v^2}{r}$ .

For conceive a body to revolve about the earth at its surface, then its centripetal force, or the force of gravity at the earth's surface, is equal to its centrifugal force. Hence by Sir I. NEWTON'S PRINCIPIA Lib. I. sect. 2. prop. 4. cor. 1. If  $R$  = the radius of the earth, and  $V$  = the velocity of a body revolving at its surface, the force of gravity  $: \frac{V^2}{R} ::$  the centripetal, and consequently the centrifugal, force of the body revolving in the circle whose radius  $= r$  with the velocity  $v : \frac{v^2}{r}$ ; but  $\frac{V^2}{R} = 32\frac{1}{2}$  feet, hence if the first term be made equal to the second, the third will be equal to the fourth.

79. To find the time in which a pendulum describes a conical surface  $CAD$ .

Let  $AB$  (fig. 2.) be the altitude, produce  $BC$  to  $e$ , and let  $Ce$  represent the centrifugal force by which the pendulum endeavours to recede from  $B$ ; draw  $Cf$  perpendicular to the horizon, and let it represent the force of gravity; compound these forces into  $Cg$ , which we will suppose to lie in the direction  $CA$ , and then the pendulum will manifestly be kept in that position. Put  $m = 32\frac{1}{8}$  feet, and let it represent the force of gravity,  $v$  = the velocity of the pendulum in the circle  $CmDn$ , then by the lem.  $\frac{v^2}{BC}$  = the centrifugal force; hence  $\frac{v^2}{BC}$ :

$m :: Ce : Cf :: CB : AB$ , consequently  $v = BC \times \sqrt{\frac{m}{AB}}$  the velocity of the pendulum in a second. Put  $p = 6,283$  &c. then  $p \times BC$  = the circumference of the circle  $CmDn$ ; hence  $BC \times \sqrt{\frac{m}{AB}} : p \times BC :: 1'' : p \times \sqrt{\frac{AB}{m}}$  the time of describing the conical surface.

Cor. 1. Hence the time of a revolution  $p \times \sqrt{\frac{AB}{m}} : \sqrt{\frac{4AB}{m}}$  the time of descent through  $2AB :: p : 2 ::$  the circumference of a circle : its diameter.

Cor. 2. Hence if the altitude be given, the time of a revolution will be given. If  $AB = 9,735$  in. the time of a revolution  $1''$ .

Cor. 3. Hence if the angle  $CAD$  be indefinitely small, the time of describing the conical surface is equal to twice the time in which the pendulum would vibrate through the diameter  $CD$ .

If in this case  $CA = 3,245$  feet, the time of revolution =  $2''$ .

#### DEFINITION.

If a circle roll upon a straight line, any point of its periphery will describe a curve called a *cycloid*.

Lem. 1. If a circle be described upon the axis of a cycloid, and an ordinate be drawn from the axis parallel to the base, the part of the ordinate intercepted between the circle and cycloid will be equal to the chord of the circle drawn from the point where the ordinate cuts the circle to the vertex.

2. The abovementioned chord of the circle is parallel to a tangent to the cycloid at the point where the ordinate meets it.

3. The cycloidal arc intercepted between the vertex and the point where the ordinate meets it, is double to the chord of the circle mentioned in lem. 1.

80. If two equal semicycloids be joined at their base



base and have their vertices downwards and axes vertical, and a pendulum equal to the length of one of them be suspended from the point where they touch and vibrate between them, it will describe a cycloid.

Mr. Atwood, in his Syllabus of a Course of Lectures read in this University, observes, that this is true only upon supposition that the whole mass of the pendulum is concentrated in a point, for it cannot otherwise take place, because as the string varies in its length the center of oscillation of a body of any magnitude will vary. The property therefore of a pendulum thus vibrating, that it performs all its vibrations in the same time, is not true, and therefore it cannot be so far a true measure of time. Pendulums therefore vibrating in circular arcs are now always used, for the same arcs will always be described in the same time. One principal source of error in a pendulum thus vibrating is, that the different temperatures of the air will alter the length of the pendulum. To prevent this, Mr. HARRISON invented a pendulum composed of rods of iron and brass so framed together, that the brass expands upwards whilst the iron expands downwards, and by thus counteracting each other, the length of the pendulum is preserved very nearly the same in all temperatures. Different methods have also been invented to answer the same purpose.

81. If a pendulum describe any arc of a cycloid, its velocity at any point varies as the right sine of a circular arc, whose diameter is the arc of the cycloid described, and versed-sine the space passed over.

82. The accelerating force of a pendulum vibrating in a cycloid varies as the arc of its distance from the lowest point.

83. The time in which a pendulum vibrates in a cycloid : the time a body would descend down the axis :: the circumference of a circle : its diameter.

84. If  $L$  be, the length of a pendulum, and  $F$  the force of gravity, the time  $T$  of vibration varies

as  $\sqrt{\frac{L}{F}}$ .

Cor. Hence all the vibrations in the same cycloid are performed in the same time.

In our latitude a pendulum 39,2 inches long is found to vibrate in one second; hence if the force be given, as it will be for the same lati-

tude, we have  $\sqrt{39,2} : \sqrt{L} :: 1'' : T'' = \sqrt{\frac{L}{39,2}}$  the time of vibra-

tion of a pendulum whose length is  $L$ ,  $L$  being taken in inches. If gravity be represented by unity, and  $F$  represent any other force acting

on the pendulum, then  $\sqrt{\frac{39,2}{1}} : \sqrt{\frac{L}{F}} :: 1'' : T' = \sqrt{\frac{L}{39,2 \times F}}$

the time of vibration. This may be applied to compare the vibrations of pendulums on different planets,  $F$  being taken to represent the respective forces of gravity on their surfaces. At the distance of  $n$  radii

from the earth's surface  $F = \frac{1}{n^2}$ , and  $T = n \sqrt{\frac{L}{39,2}}$ ; and at the dis-

tance of  $\frac{1}{n}$  of the radius within,  $F = \frac{1}{n}$ , and  $T = \sqrt{\frac{n \times L}{39,2}}$ . Hence if

$n$  be infinite, or  $\frac{1}{n} = 0$ ,  $T$  is infinite; for in this case, as the point of sus-

pension is at the center of the earth, the direction of gravity is in the direction of the pendulum, and therefore if the pendulum be made to vibrate it must continue to revolve in a circle.

85. If  $L$  be the length of a pendulum, and  $D : C ::$  the circumference of a circle : its diameter, then the space  $S$  through which a body falls in the time of vibration : the time down  $\frac{1}{2}L :: C^2 : D^2$ .

Hence if  $S$  be given  $L$  will be known, and if  $L$  be given  $S$  will be known.

#### ON THE MOTION OF PROJECTILES.

86. If the force of gravity were constant and acted in parallel lines, and there were no resistance from the air, a body thrown in any direction would describe a parabola.

From

From the small distances to which we can project bodies upon the earth's surface, the want of an uniform gravity, and its not acting in parallel lines, will not sensibly cause the motion of the body to deviate from a parabola; but the resistance of the air is so great, particularly in swift motions, the resistance varying as the square of the velocity, and sometimes in a greater ratio, that no practical conclusions can be drawn from this theory.

87. The velocity  $V$  in any point of the parabola is that which would be acquired by a body falling down  $\frac{1}{4}$  of the latus rectum  $L$  belonging to that point.

Cor. Hence if  $m = 16\frac{1}{12}$  feet,  $V = \sqrt{mL}$ , and the time of a body's descent down  $L = \frac{V}{m}$ . Hence also  $V$  varies as  $\sqrt{L}$ . As gravity acts in parallel lines perpendicular to the horizon, the horizontal velocity of a body would not be altered if there were no resistance of the air, and hence a body in any part of its motion would strike an object perpendicular to the horizon with the same force.

88. If a body be thrown in any direction, its amplitude upon an horizontal plane varies as the sine of double the angle of elevation, its altitude varies as the versed-sine of double the angle of elevation, and the time of flight varies as the sine of the angle of elevation.

89. If  $v$  be the velocity of projection,  $m = 16\frac{1}{12}$  feet,  $s =$  the sine,  $r =$  the versed-sine of double the angle of elevation,  $t =$  the sine of the angle of elevation to radius unity,  $a =$  the amplitude on an horizontal plane,  $b =$  the altitude,  $x =$  the time of flight; then will  $a = \frac{sv^2}{2m}$ ,  $b = \frac{rv^2}{8m}$  and  $x = \frac{tv}{m}$ .

Hence the amplitude is the greatest at the angle of  $45^\circ$  and  $= \frac{v^2}{2m}$ , or  $\frac{1}{2}L$ ;



$\frac{1}{2}L$ ; and the amplitudes are equal at angles equally above and below  $45^\circ$ . This would be the case if there were no resistance from the air, but on account of that, the greatest ranges are at an angle less than  $45^\circ$ .

90. If a body be projected upon an inclined plane, the cosine of whose inclination is  $w$ ,  $r$  and  $s =$  the sines of the angles which the direction makes with the plane and zenith,  $v =$  the velocity of projection,  $a =$  the amplitude,  $b =$  the altitude,

$t =$  the time of flight; then  $a = \frac{rsv^2}{m w^2}$ ,  $b = \frac{r^2 v^2}{4 m w^2}$ ,

and  $t = \frac{rv}{mw}$ .

We have before observed, that in very quick motions the theory and practice will differ very much. Mr. ROBINS observes, that a 24 pound shot, impelled with its usual charge of powder, meets with an opposition from the air equivalent to 400 lb. which retards the motion so much, that the range at an elevation of  $45^\circ$  would not be above  $\frac{1}{5}$  of that given by theory. And by experiments made with a wooden bullet fired at  $45^\circ$ , he found the range to be only 200 yards, whereas, without any resistance, it would have gone 15000 yards. Mr. ROBINS also found that very little advantage was gained by projecting a body with a greater velocity than 1200 feet in a second. He found that a 24 lb. shot, when discharged with a velocity of 2000 feet in a second, will be reduced to 1200 feet in a second in a flight of a little more than 500 yards. In consequence of this quick destruction of velocity, he found that a less projectile velocity at the same angle might carry a ball further than a greater; for the body projected with the greater velocity, when its velocity becomes equal to that of the other projection, has a less angle of elevation, on which account it may go to a less distance from thence, so as to make the whole distance less. No gun to carry far should be charged with powder whose weight is more than  $\frac{1}{6}$  or  $\frac{1}{5}$  of the weight of the ball, for that will give field pieces a velocity of 1200 feet in a second. In a battering piece, to fire at a near object, the charge should be about  $\frac{1}{3}$  of the weight of the ball. When the velocity of the body is greater than about 1100 or 1200 feet in a second, the resistance appears to be nearly 3 times greater than it ought, if it varied only as the square of the velocity. On this Mr. ROBINS makes the following curious remark. "The velocity at which the variation of the law of resistance takes place, is nearly the same as that with which sound moves. Indeed if the treble resistance in the greater velocities is owing to a vacuum being left behind the resisted body, it is not unreasonable

reasonable to suppose, that the celerity of sound is the very least degree of celerity with which a projectile can form this vacuum, and can in some sort avoid the pressure of the atmosphere on its hinder parts. It may perhaps confirm this conjecture to observe, that if a bullet, moving with the velocity of sound, does really leave a vacuum behind it, the pressure of the atmosphere on its fore part is a force about 3 times as great as its resistance, computed by the laws observed for slow motions." Now we may here observe, that, taking with Mr. COTES the height of an homogeneous atmosphere 29254 feet, if we suppose the velocity with which the air would rush into a vacuum, is, like other fluids, that which is acquired by falling through the whole height 29254 feet of an uniform atmosphere, that velocity will be found to be 1368 feet in a second, which differs but a little from that supposed by Mr. ROBINS. That bodies are therefore resisted in air as the squares of their velocities is not true, if the velocity be greater than about 1200 feet in a second; after that the resistance is almost 3 times greater than this law gives it.

91. If a body be projected, and has a rotatory motion about an axis perpendicular to the horizon, it will deviate from a plane perpendicular to the horizon.

Mr. ROBINS was the first who confirmed this by experiment. Balls from guns are frequently found to deviate to the right or left of the direction of projection. Rifle barrels are therefore used to prevent this effect. For the deflecting power is found to act on that side where the rotatory and progressive motions conspire. Now in rifle barrels, the axis of the ball's motion lies in the plane of projection, therefore the rotatory motion being perpendicular to the progressive motion, no effect is produced. But in every other position of the axis there must be a deflection, and so much the greater by how much the more the axis deviates from the plane of projection towards the horizon.

#### ON THE MOTION OF BODIES AFFECTED BY FRICTION.

92. If one hard body move upon another, its friction will be an uniformly retarding force.

For if a body be drawn upon an horizontal plane by another hanging perpendicularly, the spaces appear to vary as the squares of the times, therefore the accelerative force must be uniform. Now without friction the acceleration would be uniform, and as it appears to be uniform with it, the difference, or retardation from the friction must be uniform.

If

If the body be covered with cloth, woollen, &c. it appears that the resistance increases with the velocity, as the space increases in a much less ratio than the square of the time.

93. If  $M$  = the moving force of the body hanging perpendicularly, expressed by its weight,  $F$  = the friction considered as equivalent to a weight without inertia drawing the body back upon the horizontal plane,  $W$  = the weight of the body upon the plane,  $S$  = the space described by  $M$  in  $t$  seconds, and  $m = 16\frac{1}{2}$  feet; then by prop. 63.

$$\frac{M-F}{M+W} \times m t^2 = S, \text{ hence } F = M - \frac{M+W}{m t^2} \times S.$$

94. The quantity of friction increases in a less ratio than the weight of the body.

If we increase  $M$  and  $W$  in the same ratio, then if  $F$  increased in the same ratio, the same space would be described in the same time; but by thus increasing  $M$  and  $W$ , it appears by experiment that  $S$  is increased in the same time, therefore  $F$  must have increased in a less ratio than  $M$  and  $W$ .

95. The friction of a body does not continue the same when it has different surfaces applied to the plane on which it moves, but the smallest surface will have the least friction.

For in the same time with the same moving force, the body describes the greatest space when it moves upon its least surface. The experiments by which these propositions were originally proved may be seen in the *Phil. Trans.* 1784.

The experiments which have been made by all authors whom I have seen, have been thus instituted. To find what moving force would *just* put a body in motion; and they have concluded from thence, that the accelerative force was then equal to the friction; but it is manifest, that any force which will put a body in motion must be *greater* than the force which opposes the motion; and hence, if there were no other objection than this, the quantity of friction could not be very accurately determined. But there is another circumstance which totally destroys the experiment, so far as it tends to show the quantity of friction, which is, the strong cohesion of the body to the plane when it lies at rest; and  
this



this is confirmed by the following experiments. First, A body of  $12\frac{1}{2}$  oz. was laid upon an horizontal plane, and then loaded with a weight of 8 lb. and such a moving force was applied as would, when the body was just *put* in motion, continue that motion without any acceleration, in which case the friction must be just equal to the accelerative force. The body was then stopped, when it appeared, that the same moving force which had *kept* the body in motion before, would not *put* it in motion, and it was found necessary to take off  $4\frac{1}{2}$  oz. from the body before the same moving force *would* put it in motion; it appears, therefore, that this body, when laid upon the plane at rest, acquired a very strong cohesion to it. Secondly, A body whose weight was 16 oz. was laid at rest upon the horizontal plane, and it was found that a moving force of 6 oz. would just *put* it in motion; but that a moving force of 4 oz. *would*, when it was just put in motion, *continue* that motion without any acceleration, and therefore the accelerative force must *then* have been equal to the friction, and not when the moving force of 6 oz. was applied.

From these experiments therefore it appears, that the cohesion was so very considerable in proportion to the friction when the body was in motion, that it must by no means be neglected in our enquiries respecting the quantity of friction. All the conclusions therefore deduced from the experiments, which have been instituted to determine the friction from the force necessary to *put* a body in motion (and I have never seen any described but upon such a principle) have manifestly been totally false; as such experiments only shew the resistance which arises from the cohesion and friction conjointly.

96. Let  $e, f, g$ , (fig. 3.) represent either a cylinder, or that circular section of a body on which it rolls down the inclined plane  $CA$  in consequence of its friction, to find the time of descent and the number of revolutions.

As it has been proved in prop. 94. that the friction of a body does not increase in proportion to its weight or pressure, we cannot therefore, by knowing the friction on any other plane, determine the friction on  $CA$ ; the friction therefore on  $CA$  can only be determined by experiments made upon *that* plane, that is, by letting the body descend from rest, and observing the space described in the first second of time; call that space  $a$ , and then, as by prop. 92. friction is an uniformly retarding force, the body must be uniformly accelerated, and consequently

the whole time of descent in seconds will be  $= \sqrt{\frac{AC}{a}}$ . Now to de-

termine the number of revolutions, let  $s$  be the center of oscillation to the point of suspension  $a$ ; then, because no force acting at  $a$  can affect the motion of the point  $s$ , that point, notwithstanding the action of the friction at  $a$ , will always have a motion parallel to  $CA$  uniformly ac-

F

celerated

celerated by a force equal to that with which the body would be accelerated if it had no friction; hence, if  $2m = 32\frac{1}{2}$  feet, the velocity acquired by the point  $s$  in the first second will be  $= \frac{2m \times CB}{CA}$ ; now the

excess of the velocity of the point  $s$  above that of  $r$  ( $r$  being the center) is manifestly the velocity with which  $s$  is carried about  $r$ ; hence the velocity of  $s$  about the center  $= \frac{2m \times CB}{CA} - 2a = \frac{2m \times CB - 2a \times CA}{CA}$ ,

consequently  $rs : ra :: \frac{2m \times CB - 2a \times CA}{CA} : \frac{2m \times ra \times CB - 2a \times ra \times CA}{rs \times CA}$

$=$  the velocity with which a point of the circumference is carried about the center, and which therefore expresses the force which accelerates the rotation; now as  $2a$  expresses the accelerative force of the body down the plane, and the spaces described in the same time are in proportion to those forces, we have  $2a : CA :: \frac{2m \times ra \times CB - 2a \times ra \times CA}{rs \times CA}$

$;\frac{m \times ra \times CB - a \times ra \times CA}{a \times rs}$  the space which any point of the circum-

ference describes about the center in the whole time of the body's descent down  $CA$ ; which being divided by the circumference  $p \times ra$

(where  $p = 6,283$  &c.) will give  $\frac{m \times BC - a \times AC}{p \times a \times rs}$  for the whole num-

ber of revolutions required.

Cor. 1. If  $a \times CA = m \times BC$ , the number of revolutions  $= 0$ , and therefore the body will then only slide; consequently the friction vanishes.

Cor. 2. Let  $a'r's'$  (fig. 4.) be the next position of  $ars$ , and draw  $tr'b$  parallel to  $sa$ , then will  $st$  represent the retardation of the center  $r$  arising from friction, and  $a'b$  will represent the acceleration of a point of the circumference about its center; hence the retardation of the center : acceleration of the circumference about the center  $:: st : a'b ::$  (by sim.  $\Delta$ 's)  $tr' : br' :: rs : ra$ .

Cor. 3. If  $a'$  coincide with  $a$ , the body does not *slide* but only *roll*; now in this case  $ss' : rr' :: as : ar$ ; but as  $ss'$  and  $rr'$  represent the ratio of the velocities of the points  $s$  and  $r$ , they will be to each other as  $\frac{2m \times BC}{CA} : 2a$  or as  $m \times CB : a \times CA$ ; hence, when the body *rolls* without *sliding*,  $as : ar :: m \times CB : a \times CA$ .

Cor. 4. The time of descent down  $CA$  is  $= \sqrt{\frac{AC}{a}}$ ; but by the

last cor. when the body *rolls* without *sliding*,  $a = \frac{m \times ra \times BC}{sa \times AC}$ , hence

the time of descent in that case  $= AC \sqrt{\frac{sa}{m \times ra \times BC}}$ ; now the time

of

of descent, if there were no friction, would be  $= \frac{AC}{\sqrt{m \times BC}}$ ; hence the time of descent, when the body *rolls* without *sliding*: time of free descent  $:: \sqrt{sa} : \sqrt{ra}$ . If the body be a cylinder, the time of rolling without sliding  $= 0,305 AC \times \sqrt{\frac{1}{BC}}$ . If  $BC = 1$  ft.  $AC = 3$  ft. 3,36 in. the time of descent  $= 1''$ . If  $AC = 4$  ft.  $BC = 4,46$  in. the time  $= 2''$ . The dimensions of the cylinder are of no consequence.

Cor. 5. By the last cor. it appears, that when the body just *rolls* without *sliding*, or when the friction is just equal to the accelerative force, the time of descent  $= AC \sqrt{\frac{sa}{m \times ra \times BC}}$ ; now it is manifest,

that the time of descent will continue the same, if the friction be increased, for the body will still freely roll, as no increase of the friction acting at  $a$  can affect the motion of the point  $s$ .

If the body be projected from  $C$  with a velocity, and at the same time have a rotatory motion, the time of descent and the number of revolutions may be determined from the common principles of uniformly accelerated motions, as we have already investigated the accelerative force of the body down the plane and of its rotation about its axis; it seems therefore unnecessary to add any thing further upon that subject.

97. Let a body be projected on an horizontal plane  $LM$  (fig. 5.) with a given velocity, to determine the space through which the body will move before it stops, or before its motion becomes uniform.

CASE I. 1. Suppose the body to have no rotatory motion when it begins to move; and let  $a$  = the velocity of projection *per second* measured in feet, and let the retarding force of the friction of the body, measured by the velocity of the body which it can destroy in one second of time, be determined by experiment and called  $F$ , and let  $x$  be the space through which the body would move by the time its motion was all destroyed, when projected with the velocity  $a$ , and retarded by a force  $F$ ; then, from the principles of uniformly retarded motion,  $x = \frac{a^2}{2F}$ ; and if  $t$  = time of describing that space, we have  $t = \frac{a}{F}$ , and hence

the space described in the first second of time  $= \frac{2a - F}{2}$ . Now it is ma-

nifest, that when the rotatory motion of the body about its axis is equal to its progressive motion, the point  $a$  will be carried backwards by the



former motion, as much as it is carried forwards by the latter; consequently the point of contact of the body with the plane will then have no motion in the direction of the plane, and hence the friction will at that instant cease, and the body will continue to *roll* on uniformly without *sliding* with the velocity which it has at that point. Put therefore  $z$  = the space described from the commencement of the motion till it becomes uniform, then the body being uniformly retarded, the spaces from the end of the motion vary as the squares of the velocities; hence

$$\frac{a^2}{2F} : a^2 (:: 1 : 2F) :: \frac{a^2}{2F} - z : a^2 - 2Fz = \text{square of the progressive}$$

velocity when the motion becomes uniform; therefore the velocity destroyed by friction  $= a - \sqrt{a^2 - 2Fz}$ ; hence, as the velocity generated or destroyed in the same time is by proposition 57. in proportion to

$$\text{the force, we have by cor. 2. prop. 96. } rs : ra :: a - \sqrt{a^2 - 2Fz} : \frac{ra}{rs}$$

$\times a - \sqrt{a^2 - 2Fz}$  the velocity of the circumference  $efg$  generated about the center, consequently  $\sqrt{a^2 - 2Fz} = \frac{ra}{rs} \times a - \sqrt{a^2 - 2Fz}$ ,

and hence  $z = \frac{rs^2 + 2rs \times ra \times a^2}{as^2 \times 2F}$  the space which the body describes before the motion becomes uniform.

2. If we substitute this value of  $z$  into the expression for the velocity, we shall have  $a \times \frac{ra}{rs}$  for the velocity of the body when its motion becomes uniform; hence therefore it appears, that the velocity of the body, when the friction ceases, will be the same whatever be the quantity of the friction. If the body be the circumference of a circle, it will always lose half the velocity before its motion becomes uniform.

CASE II. 1. Let the body, besides having a progressive velocity in the direction  $LM$ , have also a rotatory motion about its center in the direction  $gfe$ , and let  $v$  represent the initial velocity of any point of the circumference about the center, and suppose it first to be less than  $a$ ; then friction being an uniformly retarding force, no alteration of the velocity of the point of contact of the body upon the plane can affect the quantity of friction; hence the progressive velocity of the body will be the same as before, and consequently the rotatory velocity generated by friction will also be the same, to which if we add the velocity about the center at the beginning of the motion, we

$$\text{shall have the whole rotatory motion; hence therefore, } v + \frac{ra}{rs} \times a - \sqrt{a^2 - 2Fz} = \sqrt{a^2 - 2Fz}, \text{ consequently } z = \frac{a^2 \times as^2 - v \times rs + a \times ra^2}{2F \times as^2}$$

the space described before the motion becomes uniform.

2. If this value of  $z$  be substituted into the expression for the velocity,

city, we shall have  $\frac{v \times rs + a \times ra}{as}$  for the velocity when the friction ceases.

3. If  $v = a$ , then  $z = 0$ , and hence the body will continue to move uniformly with the first velocity.

4. If  $v$  be greater than  $a$ , then the rotatory motion of the point  $a$  on the plane being greater than its progressive motion and in a contrary direction, the absolute motion of the point  $a$  upon the plane will be in the direction  $ML$ , and consequently friction will now act in the direction  $LM$  in which the body moves, and therefore will accelerate the *progressive* and retard the *rotatory* motion; hence it appears, that the progressive motion of a body may be accelerated by friction. Now to determine the space described before the motion becomes uniform, we may observe, that as the progressive motion of the body is now accelerated, the velocity after it has described any space  $z$  will be  $= \sqrt{a^2 + 2Fz}$ , hence the velocity acquired  $= \sqrt{a^2 + 2Fz} - a$ , and consequently the rotatory velocity destroyed  $= \frac{ra}{rs} \times \sqrt{a^2 + 2Fz} - a$ , hence  $v - \frac{ra}{rs} \times \sqrt{a^2 + 2Fz} - a = \sqrt{a^2 + 2Fz}$ , therefore  $z = \frac{rs \times v + ra \times a^2}{2F \times \frac{ra}{rs} \times a^2}$  the space required.

5. If  $a = 0$ , or the body be placed upon the plane without any progressive velocity, then  $z = \frac{rs^2 \times v^2}{2F \times as^2}$ .

CASE III. 1. Let the given rotatory motion be in the direction  $gef$ ; then as the friction must in this case always act in the direction  $ML$ , it must continually tend to destroy both the progressive and rotatory motion. Now as the velocity destroyed in the same time is in proportion to the retarding force, and the force which retards the *rotatory* is to the force which retards the *progressive* velocity by cor. 2. prop. 96. as  $ra : rs$ , therefore if  $v$  be to  $a$  as  $ra$  is to  $rs$ , then the retarding forces being in proportion to the velocities, both motions will be destroyed together, and consequently the body, after describing a certain space, will rest; which space, being that described by the body uniformly retarded by the force  $F$ , will, from what was proved in case I. be equal to  $\frac{a^2}{2F}$ .

2. If  $v$  bear a greater proportion to  $a$  than  $ra$  does to  $rs$ , it is manifest, that the rotatory motion will not be all destroyed when the progressive is; consequently the body, after it has described the space  $\frac{a^2}{2F}$ , will return back in the direction  $ML$ ; for the progressive motion being then destroyed, and the rotatory motion still continuing in the direction  $gef$ , will cause the body to return with an accelerated velocity until

until the friction ceases by the body's beginning to roll, after which it will move on uniformly. Now to determine the space described before

this happens, we have  $rs : ra :: a : \frac{ra \times a}{rs}$  the rotatory velocity

destroyed when the progressive is all lost; hence  $v - \frac{ra \times a}{rs} =$

$\frac{v \times rs - a \times ra}{rs}$  = the rotatory velocity at that time, which being sub-

stituted for  $v$  in the last article of case II. gives  $\frac{v \times rs - a \times ra^2}{2F \times as^2}$  for the space described before the motion becomes uniform.

3. If  $v$  have a less proportion to  $a$  than  $ra$  has to  $rs$ , it is manifest, that the *rotatory* motion will be destroyed before the *progressive*; in which case a rotatory motion will be generated in a contrary direction until the two motions become equal, when the friction will instantly cease, and the body will then move on uniformly. Now  $ra : rs :: v :$

$\frac{v \times rs}{ra}$  the progressive velocity destroyed when the rotatory velocity

ceases, hence  $a - \frac{v \times rs}{ra} = \frac{a \times ra - v \times rs}{ra}$  = progressive velocity when it

begins its rotatory motion in a contrary direction; substitute therefore this quantity for  $a$  in the expression for  $z$  in case I. and we have

$\frac{rs^2 + 2rs \times ra \times a \times ra - v \times rs^2}{as^2 \times ar^2 \times 2F}$  for the space described after the rotatory

motion ceases before the motion of the body becomes uniform. Now to determine the space described before the rotatory motion was all destroyed, we have (as the space from the end of a uniformly retarded

motion varies as the square of the velocity)  $a^2 : \frac{a^2}{2F} :: \frac{a \times ra - v \times rs^2}{ra^2}$

$: \frac{a \times ra - v \times rs^2}{2F \times ra^2}$ , the space that would have been described from the

time that the rotatory velocity was destroyed, until the progressive motion would have been destroyed, had the friction continued to act; hence

$\frac{a^2}{2F} - \frac{a \times ra - v \times rs^2}{2F \times ra^2} = \frac{2av \times ra \times rs - v^2 \times rs^2}{2F \times ra^2}$  = the space described

when the rotatory motion was all destroyed; hence

$\frac{rs^2 + 2rs \times ra \times a \times ra - v \times rs^2}{as^2 \times ar^2 \times 2F} + \frac{2av \times ra \times rs - v^2 \times rs^2}{2F \times ra^2}$  = whole space

described by the body before its motion becomes uniform.



## D E F I N I T I O N.

The *center of friction* is that point in the base of a body on which it revolves, into which if the whole surface of the base, and the mass of the body were collected, and made to revolve about the center of the base of the given body, the angular velocity destroyed by its friction would be equal to the angular velocity destroyed in the given body by its friction in the same time.

## 98. To find the center of friction.

Let  $FGH$  (fig. 6.) be the base of a body revolving about its center  $C$ , and suppose  $a, b, c$ , &c. to be indefinitely small parts of the base, and let  $A, B, C$ , &c. be the corresponding parts of the solid, or the prismatic parts having  $a, b, c$ , &c. for their bases; and  $P$  the center of friction. Now it is manifest, that the decrement of the angular velocity must vary as the whole diminution of the momentum of rotation, caused by the friction *directly*, and as the whole momentum of rotation or effect of the inertia of all the particles of the solid, *inversely*; the *former* being employed in diminishing the angular velocity, and the *latter* in opposing that diminution by the endeavour of the particles to preserve in their motion. Hence, if the effect of the friction vary as the effect of the inertia, the decrements of the angular velocity in a given time will be equal. Now as the quantity of friction (as has been proved from experiments) does not depend on the velocity, the effect of the friction of the elementary parts of the base  $a, b, c$ , &c. will be as  $a \times aC, b \times bC, c \times cC$ , &c. also the effect of the inertia of the corresponding parts of the body will be as  $A \times aC^2, B \times bC^2, C \times cC^2$ , &c. Now when the whole surface of the base and mass of the body are concentrated in  $P$ , the effect of the friction will be as  $a+b+c+\&c. \times CP$ , and of the inertia as  $A+B+C+\&c. \times CP^2$ ; consequently  $a \times aC + b \times bC + c \times cC + \&c. : a+b+c+\&c. \times CP :: A \times aC^2 + B \times bC^2 + C \times cC^2 + \&c. : A+B+C+\&c. \times CP^2$ ; and hence  $CP = \frac{A \times aC^2 + B \times bC^2 + C \times cC^2 + \&c. \times a+b+c+\&c.}{a \times aC + b \times bC + c \times cC + \&c. \times A+B+C+\&c.}$  (if

$S$  = the sum of the products of each particle into the square of its distance from the axis of motion,  $T$  = the sum of the products of each part of the base into its distance from the center,  $s$  = the area of the base,  $t$  = the solid content of the body)  $\frac{S \times s}{T \times t}$ .

99. Given the velocity with which a body begins to revolve about the center of its base, to determine the number of revolutions which the body will make before all its motion is destroyed.

Let

Let the friction, expressed by the velocity which it is able to destroy in one second in the body if it were projected in a right line horizontally be determined by experiment, and called  $F$ ; and suppose the initial velocity of the center of friction  $P$  about  $C$  to be  $a$ . Then conceiving the whole surface of the base and mass of the body to be collected into the point  $P$ , and (as has been proved in prop. 97.)  $\frac{a^2}{2F}$  will be the space which the body so concentrated will describe before all its motion is destroyed; hence if we put  $z = PC$ ,  $p =$  the circumference of a circle whose radius is unity, then will  $pz =$  circumference described by the point  $P$ ; consequently  $\frac{a^2}{2pzF} =$  the number of revolutions required.

Cor. If the solid be a cylinder and  $r$  be the radius of its base, then  $z = \frac{3r}{4}$ , and therefore the number of revolutions  $= \frac{2a^2}{3prF}$ .

100. When a wheel turns upon an axle, the force to overcome the friction is diminished in the ratio of the radius of the wheel to the radius of the axle.

For the force which turns the wheel acts at the circumference of the wheel, and the friction acts at the circumference of the axle; the force therefore acting at a greater distance from the axis acts at so much a greater advantage to overcome the friction.

Hence in friction wheels, where the axle of the wheel to which the weight is applied lies upon the circumference of two other wheels turning upon their axles, the friction is diminished in the ratio of the product of the radii of the wheels to the product of the radii of the axles.

#### ON WHEEL CARRIAGES.

##### *On plain hard ground.*

101. The utility of wheels arises from their turning about their axles.

For it requires a less force to draw the carriage when they are free to turn about their axles, than when they are chained together and cannot turn.

102. If the wheels be all equal and narrow, it  
re-

requires the same weight to draw the carriage, whether it be loaded before or behind.

103. If broad wheels be put on of the same size and weight, it requires the same weight to draw the carriage as for the narrow wheels, at whatever part it is loaded.

104. If two wheels be low and two high, it requires a greater weight to draw the carriage than when all are high.

In this case it makes no sensible difference which go before. The common opinion therefore that the high wheels drive on the lower when they go forward is not true.

105. If the wheels be all equal, it requires a greater weight to draw the carriage, the less the wheels are.

The disadvantage of small wheels arises from hence, that the resistance of the ground, which turns the wheels about, more easily overcomes the friction at the axle in a large than a small wheel, because it acts at a greater distance. For the mechanical advantage of wheels is, that the resistance which must be overcome by a force more than equivalent to it if the wheels could not turn, is overcome by a less force in the proportion of the radius of the wheel to the radius of the axle, when the wheels do turn. Hence the disadvantage of laying the load upon the low wheels, as it increases the friction where there is the least power to overcome it. Dr. DESAGULIERS has given a wrong reason for the disadvantage of small wheels; for he says, that the smaller wheel moving quicker upon the axis than the large one, must have so much the more friction; whereas it appears by prop. 92. that the friction is the same whatever be the velocity. Where the load is but small, and consequently the friction but small, there is but a small difference between the small and large wheels; but when the load is great the difference becomes considerable. The high wheels used in these experiments were 6 inches diameter, and the low wheels 3, and the carriage weighed about 22 oz. exclusive of the wheels.

*On hard ground with obstacles.*

106. If  $W$  be the weight of the carriage, and the center of gravity be in the middle; also if  $r =$   

$G$

the



the radius of the wheel and  $x$  = the height of the obstacle, then the power  $P$  acting parallel to the horizon which is just sufficient to balance the carriage at the obstacle without drawing it over =

$$\frac{W \times \sqrt{2rx - x^2}}{2r - 2x}.$$

For the power may be conceived to be drawing a weight up an inclined plane which is a tangent to the circle at the point where it touches the obstacle; and as, when that end rises, the other rests upon the horizontal plane, the power has to elevate a weight only equal to  $\frac{1}{2}W$ .

Experiments of this kind are subject to inaccuracies which cannot be accounted for. The power will sometimes hang for some time without moving the carriage, and then it will suddenly draw the carriage over the obstacle. Sometimes there will be a difference of  $\frac{1}{2}$  oz. out of about 10 oz. in drawing the same carriage over the same obstacle, although every care is taken to have all the circumstances accurately the same. Many of the experiments however answer very nearly to the theory, nor do any of them differ from it very materially.

The use of high wheels in going over obstacles is very manifest from this proposition, and as carriages are continually going over obstacles, high wheels will always have the advantage. Moreover in sinking into holes they have a double advantage, first, they do not sink so deep as low ones would, and secondly, after sinking, they ascend again with less power. As, when the center of gravity is in the middle of the carriage, the power has but half its weight to elevate in going over an obstacle, therefore when the load is not in the middle, it throws the center of gravity towards one end, and therefore when that end goes over an obstacle the power has more than half the weight to raise, the pressure upon each wheel being inversely as the distance of the center of gravity from them. Hence every carriage should be loaded most towards the highest wheels, by which means less than half the weight will be thrown upon the lower wheels, and thus each pair of wheels may be made to require the same power to draw them over an obstacle. The same power however that may be necessary for one obstacle will not be sufficient for another.

If the height of the obstacle be inconsiderable in respect to the radius of the wheel, which is the case with the common obstacles, as stones,

&c. which carriages usually meet with, then  $P = W \times \sqrt{\frac{x}{2r}}$ . Now

as each pair of wheels has the same obstacles to go over,  $x$  is given, and that  $P$  may be given, or that it may require the same power for each pair,  $W$  must vary as  $\sqrt{r}$ ; now the weight supported by each wheel is inversely as its distance from the center of gravity. Hence to overcome small obstacles, the distance of the center of gravity from the

the great wheels : its distance from the small :: the square root of the radius of the small wheel : the square root of the radius of the large wheel. Now I find that the radii of the wheels of a common waggon are about 5 ft. 8 in. and 4 ft. 8 in. and the distance of the wheels, when narrow, about 6 ft. 6 in.; hence the center of gravity of the load of a waggon ought to be about 3,6 in. nearer to the higher than to the lower wheels. For a broad wheel waggon, where the distance of the wheels is about 7 ft. 10 in. the center of gravity ought to be about 4,2 in. nearer to the higher than to the lower.

It appears also that when  $W$  and  $x$  are given and  $x$  is very small,  $P$  varies inversely as the square root of the radius of the wheel. Hence the advantage of a wheel to overcome a small obstacle varies as the square root of the radius of the wheel. This resistance of the obstacle causes the wheel to turn, but this resistance is not friction; for friction arises from the rubbing of the parts of one body against those of another, whereas here the wheel only turns upon a point; the friction therefore only takes place at the axle, where the parts rub one against another. There is therefore no friction at the ground, unless when the wheels slide, which is the case when they are chained together, which is frequently done to prevent them from running too fast down a hill.

### *Upon sand.*

107. It requires a less force to draw a narrow than a broad wheel carriage upon sand.

The disadvantage of the broad wheels seem to arise from their driving the sand before them.

108. If two wheels be high and two low, it requires a greater force to draw the carriage than when all the wheels are high.

109. If all the wheels be low, it requires a greater force to draw the carriage than in the last case.

In all these cases it requires a less force to draw the carriage when loaded behind than before.

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# H Y D R O S T A T I C S.

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## D E F I N I T I O N.

1. **A FLUID** is a body whose parts are put in motion one among another by any force impressed; and which, when the impressed force is removed, restores itself to its former state.

All fluids may be divided into elastic and non-elastic; elastic comprehend the different kinds of airs, and non-elastic the different kinds of liquids. As many bodies, by cold, from liquids become solids, such bodies might be defined to be liquids so long as their surfaces, when disturbed, will restore themselves to an horizontal position. The definition supposes a partial pressure; for if the fluid be incompressible, under an equal general pressure, none of the parts will move. From the ease with which the parts of a fluid are moved, it is supposed to be constituted of particles round and very smooth.

2. The specific gravity, and also the density of a body, is in proportion to its quantity of matter or weight, when the magnitude is given.

## O N T H E P R E S S U R E O F F L U I D S.

1. A fluid weighs as much in a fluid of the same kind, as it does out of the fluid.

2. The surface of every fluid at rest is horizontal.

3. If the density of a fluid be uniform, the pressure at any depth is in proportion to the depth.

4. Fluids press equally in all directions.

Hence the lateral pressure of a fluid is equal to the perpendicular pressure. This is one of the most extraordinary properties of fluids, and can be conceived to arise only from the extreme facility with which the component particles move amongst each other. It will be difficult to conceive how this is possible to happen if we suppose the particles to be in contact; they are therefore probably kept at a distance from each



each other by a repulsive power, which power in water, admitting the truth of the experiments related, must be greater than any human power that can be applied. This is one remarkable difference between solids and fluids, as the former press only downwards.

This and the last prop. accounts for what is called the hydrostatical paradox, by which very great weights may be balanced by a very small weight of water without its acting at any mechanical advantage.

5. If two fluids of the same kind communicate, their surfaces will rest in the same horizontal plane.

The *Romans* do not appear to have known this property of fluids, otherwise they would not have been at the expence of conveying their water from one place to another over arches built one upon another.

6. If a fluid press against any surface, the pressure varies as its area multiplied into the depth of its center of gravity, the pressure at every point being estimated in a direction perpendicular to the surface at that point.

The pressures of different fluids will vary as above, and as their densities conjointly.

7. The pressure of a fluid against any surface is equal to the weight of a cylinder of that fluid, whose base is equal to the area of the surface and altitude equal to the depth of its center of gravity.

#### DEFINITION.

The center of pressure of a surface is that point to which if a force equal to the whole pressure were applied, but in a contrary direction, it would keep the surface at rest.

8. If a plane surface which is pressed by a fluid be produced to the surface of the fluid, and their common intersection be made the axis of suspension, the center of oscillation will be the center of pressure.

9. If two different fluids communicate, they will

will stand at altitudes from the plane where they meet, which are inversely as their specific gravities or densities.

If mercury and water thus communicate, the altitude of the latter will be about 14 times that of the former.

### ON THE SPECIFIC GRAVITIES, OR DENSITIES OF BODIES.

10. The weight  $W$  of a body varies as its magnitude  $M$  and specific gravity  $S$  conjointly.

A cubic foot of rain water weighs 1000 ounces avoirdupoise; call the specific gravity of this  $s$ , and then  $W : 1000 :: M \times S : 1 \text{ ft.} \times s$ , hence if we assume  $s = 1000$  as a standard to compare the specific gravities with, we have  $W = M \times S$ , where  $M$  is the magnitude in feet. To reduce it to the measure in cubic inches we have  $1728 : 1 :: 1000 : 5787$  the weight of a cubic inch of rain; hence  $W = 5787 MS$ , where  $M$  is the magnitude in cubic inches. Now a troy ounce : an avoirdupoise ounce :: 480 : 437,5, for an avoirdupoise ounce contains 437,5 grains troy. If we reduce  $W$  to troy weight, which is most commonly used, we shall have  $W = 52746 MS$  troy ounces = 253,18  $MS$  grains. Hence if  $W$  and  $S$  be given, we know  $M$  the magnitude in cubic inches. By this method Mr. Atwood, in his Analysis, constructed the following table to find the capacity of an irregular vessel.

Let the vessel be filled with water, the weight of which is  $A$  ounces, then  $52746 : A :: 1 : \text{the capacity in cubic inches.}$  Hence

| oz. | cub.in.  | oz. | cub.in.   | oz. | cub.in.   |
|-----|----------|-----|-----------|-----|-----------|
| 1   | = 1,8959 | 4   | = 7,5835  | 7   | = 13,2712 |
| 2   | = 3,7918 | 5   | = 9,4794  | 8   | = 15,1671 |
| 3   | = 5,6877 | 6   | = 11,3753 | 9   | = 17,0630 |

Hence we have a very accurate method of determining the diameter  $d$  of any sphere whose weight is  $w$  and specific gravity  $s$ , that of water being unity. For the solid content of a sphere whose diameter is 1 is ,5236; hence  $1 : ,5236 :: 253,18 \text{ grains} : 132,428 \text{ grains}$  the weight of a sphere of water whose diameter is 1; hence, as the weights of spheres are as their specific gravities and cubes of their diameters con-

jointly, we have  $132,428 s d^3 = w$ , consequently  $d = \sqrt[3]{\frac{w}{s}} \times ,19612$ ,  
*or more accurately*  $d = \sqrt[3]{\frac{w}{s}} \times ,1961083$  *or by logarithms*  $d =$   
 which is the rule given by Mr. Atwood.  
 $\frac{1}{3} \times \log. w - \log. s + 9,2926731$

11. If a body swim on a fluid, it will not rest till its center of gravity is in a vertical line with the center of gravity of the water displaced.

For

For the body is supported by a force under it equal to the reaction of the fluid displaced, and the effect of the force of this fluid and of the body supported being the same as the effect of each concentrated in its center of gravity, their centers of gravity must be in the same vertical line, otherwise the body will not be supported.

12. If a body swim on a fluid, it displaces as much fluid as is equal in weight to the body; and the part immersed : the whole body :: the specific gravity of the body : that of the fluid.

The *HYDROMETER* is an instrument for finding the specific gravities of fluids, and is constructed upon the principle of this proposition, by measuring how far it sinks in different fluids, and the parts immersed are inversely as the specific gravities of the fluids. It is usually a brass stem with a bubble at the bottom into which something heavy is put to make it sink and keep the stem, which is graduated, upright, in order to shew how much it sinks in different fluids; and by knowing the weight of the whole instrument and of any part of the stem, you determine their specific gravities. This instrument is also made use of to find whether a liquor is above or below proof, by observing whether it stands above or below that point upon the stem which is proof. Many improvements have been made to this instrument, of which those lately made by Mr. RAMSDEN seem to be the most important.

In this proposition we neglect the effect of the air upon the part without the fluid, which is considered in prop. 17.

13. The weight which a body loses when wholly immersed in a fluid is equal to the weight of an equal bulk of the fluid.

14. The weight which a body loses in a fluid : its whole weight :: the specific gravity of the fluid : that of the body.

If the body which is weighed in a fluid be wood, it should first be well rubbed over with grease, or be varnished, to prevent its imbibing any of the fluid.

When we say a body loses part of its weight, we do not mean that it gravitates less than it did before, or that its real weight is less, but that it is partly supported by the reaction of the fluid upwards against its under surface, and therefore it requires a less weight to support it; the weight thus said to be lost is communicated to the fluid, for the sum of the weights of the body and fluid is just the same when the body is in the fluid as when out. By this proposition the specific gravity of a solid and fluid are compared.



Cor. 1. Hence if different bodies be weighed in the same fluid, their specific gravities will be as their whole weights directly and the weights lost inversely.

If the body to be examined consist of small fragments, they may be put into a small bucket and weighed, and then if from the weight of the bucket and body in the fluid we subtract the weight of the bucket in the fluid, there remains the weight of the body in the fluid.

Cor. 2. If the same body be weighed in different fluids, their specific gravities will be as the weights lost.

The body for this purpose should not be wood, as that would im-  
bibe the fluids; a solid glass bulb is the most proper.

Cor. 3. Hence if two bodies of different magnitudes balance each other in any fluid, the greater will preponderate in a lighter fluid, and the less in an heavier.

15. If  $Q$  be a body lighter than the fluid, connect it with another  $P$  which is heavier so that together they may sink, and let the weight of  $P$  in the fluid be  $a$ , and the weight of  $P+Q$  in the fluid be  $b$ , and let  $d$  be the weight of  $Q$  out of the fluid; then the specific gravity of  $Q$  : the specific gravity of the fluid  $:: d : a - b + d$ .

16. A body immersed in a fluid, ascends or descends with a force equal to the difference between its own weight and the weight of an equal bulk of the fluid.

17. If a lighter fluid rest upon an heavier, and their specific gravities be as  $a : b$ , and a body whose specific gravity is  $c$  rest with one part  $P$  in the upper fluid and the other part  $Q$  in the lower, then  $P : Q :: b - c : c - a$ .

Cor. Hence  $Q : P+Q :: c - a : b - a$ ; and if  $a$  be so small that it may be neglected,  $Q : P+Q :: c : b$ , as in prop. 12.

18. If  $a$  and  $b$  be the specific gravities of two fluids to be mixed together,  $P$  and  $Q$  their magnitudes, and  $c$  the specific gravity of the compound, then  $P : Q :: b - c : c - a$ .

It

It is here supposed that the magnitude of the two fluids when mixed is equal to the sum of the two magnitudes when separate. But it very often happens that the magnitude of the mixture is less than this sum, owing, probably, partly to the constituent particles of the different fluids being of different magnitudes, and partly to their chemical attraction. This is called a penetration of dimensions. If water and oil of vitriol be mixed together, the magnitude of the mixture is less than the sum of the two magnitudes when separate. ✓

19. The ascent of a light body in an heavier fluid arises from the pressure of the fluid upwards against its under surface. *by Art 4.*

For if the body be placed upon the bottom of the vessel, and so closely fitted to it that no part of the fluid can get under, it will rest; but if it be lifted up so that the fluid gets under it, it immediately rises.

20. If a plate of brass be closely fitted to the bottom of a glass vessel of the same size, and then immersed in a fluid, when it is sunk to about 8 times the depth of its thickness, it will then be supported by the fluid under it. *Atwood.*

#### ON THE TIME OF EMPTYING VESSELS, AND ON SPOUTING FLUIDS.

21. If a fluid spout from the bottom or side of a vessel, at a small distance from the orifice the stream is contracted, and when the orifice is small, the diameter of the smallest part : the diameter of the orifice :: 21 : 25, as determined by Sir I. NEWTON.

When a fluid spouts from a vessel, the water from all the sides rushing towards the orifice is the cause of the contraction of the stream, called the *vena contracta*. Now the area of the orifice : the area of the smallest section of the stream ::  $25^2 : 21^2$  which is very nearly as  $\sqrt{2} : 1$ ; hence, as the velocity is inversely as the area of the section, the velocity at the vena contracta : the velocity at the orifice ::  $\sqrt{2} : 1$ . Now from the quantity of water running out in a given time, and the area of the vena contracta, Sir I. NEWTON found that the velocity there was that which a body acquires in falling down the altitude of the fluid above the orifice; hence the velocity at the orifice being less in the

H

ratio

ratio of  $\sqrt{2} : 1$ , must be that which is acquired in falling down half the altitude. We must therefore distinguish between the velocity at the orifice and at the *vena contracta*; and in the doctrine of spouting fluids, it is the latter velocity which we must consider, and assume the point of projection from that point. This is true only upon supposition that the orifice is very small; if not, the velocity there is less than that acquired in falling down half the height. It appears however from some experiments which I have made, that the velocity of the effluent water is in no *constant* ratio of any part of the depth; when the orifice is very small it appears that the velocity there is very nearly that which is acquired in falling down half the depth; but as the surface of the fluid descends the velocity is greater than that acquired through half the depth. See PARKINSON'S Hydrostaticks, p. 83.

22. If a vessel empty itself through an orifice at the bottom, and the area of the section parallel to the bottom continue the same, the velocity of the surface of the fluid is uniformly retarded.

23. The velocity of a fluid through different sections of the same tube or vessel varies inversely as the area of the section.

If  $a$  = the surface of a fluid running out of an orifice  $o$  at the bottom of a vessel,  $x$  = the depth of the fluid at that time,  $t$  = the time the fluid has been running; then  $t = \frac{a}{o} \times \frac{-x}{\sqrt{2sx}}$ , where  $s = 16\frac{1}{2}$  feet.

Cor. 1. If the vessel be cylindrical or prismatic, and  $h$  = the altitude, then  $t = \frac{a}{o} \times \sqrt{\frac{2h}{s}}$  the time of emptying.

Cor. 2. The time of the descent of the fluid in the same vessel through any space  $z = \frac{a}{o\sqrt{s}} \times \sqrt{2h - \sqrt{2h - 2z}}$ .

If the depth of a cylindrical vessel be 12 in. and the diameter of the orifice at the bottom be 0,3 inches, the time of emptying by theory = 35", which agrees very nearly with experiment.

24. If a cylindrical vessel be kept constantly filled with a fluid, twice the quantity which the vessel contains will run out in the time it would have emptied itself.

This appears by mechanics prop. 60. because the surface of the fluid, when the vessel empties itself, is uniformly retarded.

In



In all the propositions respecting the times in which vessels empty themselves, the orifice is supposed to be very small in respect to the bottom of the vessel, in which case our theory and experiments agree very nearly; otherwise they do not. This agreement of the theory with experiment, proves the velocity at the orifice, whose magnitude is under the above restriction, to be that which a body acquires in falling down half the depth of the fluid, the theory being founded upon that principle. As the orifice increases, the velocity is that which is acquired in falling down less than half the depth.

25. If a cylindrical vessel 12 inches high have a circular orifice at the bottom of 0,3 in. diameter, it will empty itself in about 35"; but if a pipe of the same length and diameter be fixed in the bottom, and the fluid flow through it, the vessel will be emptied in about 18", whether, when you fill the vessel, you stop the tube at the top or the bottom; but if the pipe increase downwards, and the diameter of the lower end be 0,42 in. the time of emptying will be about 13".

26. If a cylindrical vessel 18 in. deep have a circular hole at the bottom, and at the side close to the bottom, of 0,45 in. diameter; also if a cylindrical pipe 13,7 in. long and of the same diameter be fixed in the side by the bottom in an horizontal position; in all these cases the time of emptying is about 25"; but if the pipe increase in its diameter, so that the diameter of the other end be 0,65 in. the time of emptying is about 16".

27. If a vessel be filled with a fluid, and from any point it be directed to spout upwards, it rises nearly to the height of the surface of the fluid in the vessel.

Two causes prevent it from rising quite so high, one is the resistance of the air, and the other is the falling back of the fluid. Hence, in

consequence of this latter obstruction, the fluid will rise higher if the direction deviate a little from the perpendicular. In great velocities the resistance is so great, that GRAVESANDE says the greatest height to which water can thus be projected is not above 100 feet.

28. The distance to which a fluid spouts from the side of a vessel upon an horizontal plane, is equal to twice the corresponding ordinate of a circular arc whose diameter is equal to the distance of the surface of the fluid from the plane.

From the distance to which a fluid spouts from the side of a vessel, it appears that the velocity with which it spouts must be that which is acquired in falling down the whole depth of the fluid above the orifice. But it has been shown that the velocity through the section of the orifice is that which is acquired in falling through half the depth of the fluid. Hence as soon as the fluid gets out of the orifice, it acquires an increase of velocity in the ratio of  $\sqrt{2} : 1$ . The pressure of the fluid in the vessel must therefore continue to act upon the effluent fluid for a small distance after it has passed the section of the orifice.

#### ON THE ATTRACTION BETWEEN SOLIDS AND FLUIDS.

29. If two glass planes, forming a very small angle with each other, be dipped in water, the water will rise between them, and the height will be greater the nearer it is to the concurrence of the planes. Mercury will in like manner sink.

The water is supposed to ascend by the attraction of the glass which lies contiguous to the surface of the fluid. The mercury descends, being more attracted by itself than by the glass. The upper surface of the fluid will form the common hyperbola, one of whose asymptotes is the common intersection of the planes, and the other lies in the surface of the fluid in the vessel. Where the distance is about  $\frac{1}{100}$  of an inch, the water rises about an inch. If the planes be parallel, the height is every where the same. Mr. HAUKEBEE tried the same with *marble* and *brass* planes, and found that the water rose between them.

30. If very small glass tubes be dipped in water, the water will stand in them, above the level of that in the vessel, at altitudes which are inversely as their diameters.

Hence

Hence the product of the diameter and altitude is a constant quantity, which quantity = ,053 of an inch. To measure the diameter of a capillary tube, put in some mercury whose weight is  $w$ , and let it occupy a length of the tube  $l$ ; then if 13,6 be the specific gravity of mercury (which it is when purest) when that of water is 1, the dia-

meter =  $\sqrt{\frac{w}{l}} \times ,01923$ . For if  $d$  = diameter, the content of the mercury =  $d^2 l \times ,7854$ , and as one cubic inch of mercury weighs 3443

grains, we have  $1 : 3443 :: d^2 l \times ,7854 : w$ , hence  $d = \sqrt{\frac{w}{l}} \times$

,01923. This rule is given by Mr. ARWOOD in his Analysis, p. 3.

It has been generally supposed that the water is suspended by the attraction of a small annular surface on the inside of the tube, contiguous to the surface of the water. But Dr. HAMILTON (Lect. 2. p. 47.) supposes it to arise from the attraction of the annulus lying just within the lower orifice of the tube. Mr. PARKINSON however, in his System of Hydrostatics, thinks neither of these suppositions true, but supposes that the fluid is sustained by the immediate attraction of the glass. As the fluid will rise in an exhausted receiver, it cannot in any way be owing to the air.

31. If  $a$  be the altitude at which water will stand in a capillary tube whose diameter is  $d$ , then if another tube whose lower part is larger but whose diameter is  $d$  at the altitude  $a$  has its air drawn out, the water will rise and be suspended at that altitude.

In this case it is supposed that the attraction of the annular surface contiguous to the surface of the water supports all the fluid immediately under that part of the tube, and the other part is supported by the pressure of the air on the surface of the fluid without the tube. That this is true appears from hence, that the experiment will not succeed in a well exhausted receiver. Dr. JURIN tried it in an exhausted receiver and says that it did succeed; this therefore must have happened from his air pump not exhausting sufficiently. Dr. HOOK makes the greatest altitude in the finest capillary tube about 21 inches.

32. If a glass tube be filled with ashes rammed very close, and the lower end be dipped in water; the water will gradually rise up through the ashes.

Mr. HAUKEBEE took a tube 32 inches long, and found that the water ascended to the top in 130 hours. He then asks, "does not this arise from the same cause as in small tubes, or between two planes?"  
and



and do not the particles of this matter, by their little hollows and intervals, form a congeries of minute slender pipes, or surfaces near each other, so that the liquor rises by one and the same cause?" It rises faster in vacuo than in the open air, because in the latter case it has all the air to force out. Water will also rise in salt, sugar, &c. in the same manner. The ascent of juices in vegetables, and the various secretions of fluids through the glands of animals, arise from the same cause, the power of attraction.

33. If the lower of two glass planes forming a very small angle with each other be parallel to the horizon, and their surface be moistened with oil of turpentine, oranges, &c. a drop of oil placed between them will move towards their concurrence.

#### ON THE RESISTANCE OF FLUIDS.

34. If a body move in a resisting medium, the resistance, within certain limits of the velocity, varies very nearly as the square of the velocity.

The resistance arises from three causes, the inertia, the tenacity, and the friction of the fluid. The latter cause is inconsiderable, and the second is, in most fluids, but very small when compared with the inertia. The resistance arising from the tenacity will be diminished as the velocity increases, but that arising from the inertia will be, within certain limits, as the square of the velocity. In very swift motions, the resistance of the air increases in a greater ratio, for the reason explained in prop. 90. of mechanics; and in other fluids the same consequence would follow, for the same reason, for projected bodies. But when bodies descend in fluids such as water, the resistance is always very nearly as the square of the velocity, because the body never can acquire a velocity beyond a certain limit. The greater the velocity is the less will be the pressure against the back part of the body, and this variation of pressure will cause a deviation in the law of resistance. Air, water and mercury are called perfect fluids, not having any sensible tenacity or friction. This doctrine of resistance was established by Sir I. NEWTON, by a variety of experiments. See the PRINCIPIA, Vol. II. prop. 31. Scholium.

35. When plane bodies move in resisting mediums, the resistance varies as their areas, squares of their velocities and densities of the mediums conjointly.

36. If

36. If a plane body move obliquely in a resisting medium, and the force of the oblique stroke : the force of the direct stroke :: the sine of the angle at which the plane strikes the fluid : radius, the resistance perpendicular to the plane varies, *cæteris paribus*, as the square of the sine of the angle at which the plane strikes the fluid.

The principle upon which this proposition is founded is not true when the body moves in air, as will be shown by experiment. I have not yet had an opportunity of examining whether it be true for water. The two next propositions suppose the truth of the same principle, and therefore are not true for air.

37. The resistance of the same plane in the direction of its motion varies as the cube of the sine of the same angle.

38. The resistance of the same plane in a direction perpendicular to its motion varies as the square of the sine of the same angle into its cosine.

Hence if we suppose the plane to be at rest and the fluid to move against it, the effect of the fluid to move the plane in a direction perpendicular to the motion of the fluid will vary in the same ratio. This, if the above principle were true, would be the effect of the wind to put the sails of a mill in motion. After they have begun their motion, the effect will depend upon the relative velocity of the sails and wind; the angle also at which the wind acts upon the sail will vary with the velocity of the sail.

39. If a plane body move in water in a direction which is perpendicular to its surface, it is resisted by a force equal to the weight of a column of the fluid whose base is equal to that surface, and height equal to that through which a body must fall to acquire the velocity of the body.

See PARKINSON'S *Hydrostatics*, p. 173. and ATWOOD on *Rectilinear and Rotatory Motion*, p. 124. This prop. is capable of a very satisfactory proof by experiment.

40. If

40. If a sphere and end of a cylinder of the same diameter moving in the direction of its axis, move in a fluid with equal velocities, the resistance of the cylinder will, by theory, be double that of the globe.

In the demonstration of this prop. the principle in prop. 37. is assumed as true. The prop. therefore is not true if the globe move in air.

#### DEFINITION.

If a plane body revolve in a resisting medium about an axis by means of a weight hanging from it, that point into which if the whole plane were collected it would suffer the same resistance, I call the *center of resistance*.

41. If  $a$  be the area of the plane, and  $a'$  the fluxion of the area at the distance  $x$  from the axis, then the distance  $d$  of the center of resistance from

$$\text{the axis} = \sqrt[3]{\frac{\text{flu. } x^3 a'}{a}}.$$

For the effect of the resistance of  $a'$  to oppose the weight must vary as the resistance into its distance from the axis, or (because the resistance varies as the square of the velocity, or as the square of the distance from the axis,) as  $x^3 a'$ , and therefore the whole resistance is as the fluent of  $x^3 a'$ . For the same reason, the effect of the resistance of  $a$  at the distance  $d$  is as  $d^3 a$ ; hence  $d^3 a = \text{flu. } x^3 a'$ , therefore  $d =$

$$\sqrt[3]{\frac{\text{flu. } x^3 a'}{a}}.$$

If the body be a parallelogram, two of whose sides are parallel to the axis and at the distances  $m$  and  $n$ ,  $m$  being the least distance, then  $d =$

$$\sqrt[3]{\frac{n^4 - m^4}{4n - 4m}} = \sqrt[3]{\frac{n^2 + m^2 \times n + m^2}{4}}.$$

42. If a plane body revolve about an axis by a moving force acting thereon, first with its edge forward and then with its plane side, the difference of the accelerative forces which will preserve the motion of the body in each case uniform with the same velocity, diminished in the ratio of the distance of the center of resistance from the center of the

the



the axis to the radius of the axis, gives the absolute resistance of a plane equal to the body moving with the velocity with which the center of resistance moves.

The machine constructed for this purpose is an horizontal axis, at one end of which there are four arms, like the sails of a mill, and at each extremity a plane is fixed. Two lines are wound round the axis together, and one leaves the axis above and the other below it, and go in opposite directions horizontally and pass over pulleys, and equal weights are hung on at each end to give motion to the axis. By this means, all pressure upon the axis from the weights is avoided, so that whatever change it may be necessary to make in the weights to give the same velocity to the planes going flat and edge ways, there is not more pressure upon the axis, and consequently not more friction. The difference of the weights therefore can only arise from the resistance of the planes moving edge and flat ways; and a weight at the center of resistance equivalent to that difference, must, from the common property of the lever, be to that difference as the radius of the axis to the distance of that center from the center of the axis. To render the resistance still greater, four arms might be put upon the other end of the axis. In my machine the radius of the axis is 0,199 in. the length of the arms 30 in. each plane is a square whose side is 4 in. one side of which is parallel to the arm, and the center of the squares is 32 in. from the axis. Hence the distance of the center of resistance from the axis = 32,044 in. The radius of the axis was determined in the following manner. A fine silk thread 36 in. long was all wound round the axis so that the whorls all touched each other, and the number of revolutions and parts of a revolution were observed; then dividing 36 in. by the number of revolutions and parts of a revolution, it gave the circumference of the axis, from which the radius was determined. By this method the radius of any cylinder may be found to a very great degree of accuracy, and the longer the string the greater the accuracy. As the string by winding runs along the axis, it does not go in a plane perpendicular to the axis, and therefore there will be a small inaccuracy from that, but it is so small that there can scarce be a case where it may be necessary to consider it. If there should, it may be corrected thus. From the square of the length of the string subtract the square of the space it takes upon the axis, and the square root of the difference is the quantity to be divided by the number of revolutions in order to get the circumference. The mean of 7 experiments gave the resistance = 0,0462 oz. troy to 64 square inches striking the air perpendicularly with a velocity of 2 feet in a second

43. If a plane body strike the air obliquely, the effect of the direct stroke is not to that of the oblique, as radius to the sine of the angle of incidence.

This

This was discovered by inclining the planes at different angles, and observing what weights would give them the same uniform velocity. The angles at which I first tried the experiment gave the force of the stroke greater in proportion than the sine of inclination. I communicated to Dr. HUTTON what I had done upon this subject, knowing that he had made experiments on resistances of the air upon a very extensive scale; when he was so obliging as to inform me, that having made experiments of the same kind at all angles of obliquity from  $0^\circ$  to  $90^\circ$ , (which I had not then done,) he found that in one part of the quadrant the force was greater and in the other part less, than in proportion to the sine of inclination, and that the variation followed a very extraordinary law. He finds the resistance of a globe  $2\frac{2}{3}$  greater than the resistance of a cylinder, moving in the direction of its axis with the same velocity. It is hoped that he will soon communicate to the public his valuable experiments.

44. The resistance of the air to plane bodies varies, *cæteris paribus*, as the square of the velocity.

For it requires four times the above mentioned difference of weights to be hung from the axis to give the planes twice the velocity.

#### ON THE MOTION OF BODIES IN FLUIDS.

45. If a sphere whose weight is  $w$  and diameter  $d$  move in a resisting medium with a velocity which a body would acquire in falling in a vacuum through the space  $b$ , and the density of the medium : the density of the body ::  $1 : b$ ; then the

resistance of the sphere  $= \frac{3wb}{4db} = \frac{pd^2b}{8}$ , where  $p =$

3,14159 &c.

See ATWOOD'S Treatise on Rectilinear and Rotatory Motion, p. 130; also PARKINSON'S Hydrostatics, p. 177.

46. The gravity of the same sphere in the fluid, being equal to its own weight diminished by the weight lost, is equal to  $w - \frac{w}{b}$ , the gravity of the body itself in vacuo being  $w$ .

47. The

47. The force with which the same sphere descends in the fluid, being equal to its gravity in the fluid diminished by the resistance, will be  $w -$

$$\frac{w}{b} - \frac{3wb}{4db}.$$

48. If  $v =$  the velocity acquired by the same sphere in descending from rest through the space  $x$ , then by mechanics prop. 72.  $v\dot{v} = 2mF\dot{x} =$

$$w - \frac{w}{b} - \frac{3wb}{4db} \times 2m\dot{x}.$$

If  $e = 2,71828$  the number whose hyp. log. is 1, then if the fluent be taken and properly corrected, we have  $\dot{v} = \sqrt{\frac{16dm}{3} \times \frac{1}{b-1} \times}$

$\sqrt{\frac{-3x}{1-e^{4bd}}}$ ; see PARKINSON'S Hydrostatics, p. 186. and ATWOOD on Rectilinear and Rotatory Motion, p. 140.

If  $x$  be increased sine limite,  $e^{\frac{-3x}{4bd}} = 0$ ; hence  $v = \sqrt{\frac{16dm}{3} \times \frac{1}{b-1}}$  the limit of the velocity which the body can acquire, but which it can

never attain. Or if  $d$  be very small when compared with  $x$ ,  $e^{\frac{-x}{4bd}}$  will be very nearly  $= 0$ ; and hence if  $d$  be very small,  $x$  may be small and

$e^{\frac{-3x}{4bd}}$  will  $= 0$  very nearly. Hence, as Mr. ATWOOD observes, very small bodies descending in a fluid very soon acquire, as to sense, their greatest velocity, and then they appear to descend uniformly. He computes that if  $x = 16d$ , the velocity, after the sphere has descended 16 diameters, will be within less than  $\frac{1}{800}$  part of the greatest velocity. Hence when a metal is dissolved in a menstruum, the particles being extremely small, will descend with so very small a velocity that they will for a long time, as to sense, appear quiescent. Mr. MACKBRIDE supposes that fixed air is the cementing principle of bodies; and Mr. ATWOOD supposes that when a body is dissolved in a menstruum, the fixed air escaping carries up the disunited parts of the body; and when the medium is once filled with the particles, they will, as is shown above, remain suspended for a very long time. The dissolution of the body is supposed to arise from hence, that the particles of the medium attract the particles of the body with a greater force than the particles of the body attract each other.



49. If  $t$  be the time in which the sphere in the last prop. descends through the space  $x$ , then  $t =$

$$\sqrt{\frac{b^2 d}{3m \times b - 1}} \times \text{hyp. log.} \frac{1 + \sqrt{1 - e^{\frac{-3x}{4bd}}}}{1 - \sqrt{1 - e^{\frac{-3x}{4bd}}}}.$$

See ATWOOD on Rectilinear and Rotatory Motion, p. 150. and PARKINSON'S Hydrostatics, p. 189.

This is upon supposition that the resistance varies as the square of the velocity; and the experiments compared with the theory agree sufficiently to establish the truth of the hypothesis. A small difference must necessarily arise, granting the supposition to be true, from the unavoidable errors in constructing the experiments.

In our experiments  $d = 2,0833$  in.  $b = 1,01014$ ,  $x = 55,5$  in.; hence  $t = 12,75$  the time by theory. The time by experiment is about  $13''$ .

The diameter was determined by the rule in prop. 10. by loading the sphere till it was of the same specific gravity as water, in which case its weight was 1198,814 grains; hence  $s = 1$ , and its diameter =

$$\sqrt[3]{1198,814} \times ,19612 = 2,0833 \text{ inches.}$$

50. As a body descends in a fluid, it continually adds more weight to the fluid until it has acquired its greatest velocity, at which time the weight added to the fluid is just the same as if the body were laid at the bottom of the vessel.

For as the velocity of the body keeps increasing, the action of the body upon the fluid will keep increasing; and when the body has acquired its greatest velocity, the resistance being equal to the weight of the body in the fluid, the body acts against the fluid with its relative weight just as it would act against the bottom of the vessel if it were laid upon it.

#### ON ELASTIC FLUIDS, AND THE DENSITY OF THE AIR AT DIFFERENT ALTITUDES.

51. If the particles of an elastic fluid repel each other with forces varying inversely as the  $n^{\text{th}}$  power of their distance, then if  $r$  represent the distance of  
the

the particles,  $d$  the density and  $c$  the compressive force,  $c$  varies as  $d^{\frac{n+2}{3}}$ .

Cor. It appears by experiment, that in the common atmospheric air the compressive force varies as the density. Hence  $\frac{n+2}{3} = 1$ , consequently  $n = 1$ , and therefore the particles of air repel each other with forces which vary inversely as their distance.

As the compressive force is equal to the elastic force of the air, action and reaction being equal and contrary, the elastic force must vary as the density.

52. If we suppose the force of gravity to vary as the  $n^{\text{th}}$ . power of the distance from the earth's center,  $r$  the radius of the earth,  $x$  any distance from the center, and  $v$  the corresponding density, that at the earth's surface being unity, and  $d$  be the height of an homogeneous atmosphere; then  $d$

$$\times \text{hyp. log. } v = \frac{r}{n+1} - \frac{x^{n+1}}{n+1 \times r^n}.$$

Cor. 1. If we suppose the force of gravity to vary inversely as the square of the distance, then  $n = -2$ ; hence  $d \times \text{hyp. log. } v = \frac{r^2}{x} - r$ .

Hence if  $x$  be taken in musical progression,  $\frac{r^2}{x}$ , and consequently  $\frac{r^2}{x} - r$ , will be in arithmetical progression, therefore the hyp. log.  $v$  will be in arithmetical progression, and hence  $v$  will decrease in geometrical progression.

Cor. 2. If we suppose gravity to be constant, then  $n = 0$ , therefore  $d \times \text{hyp. log. } v = r - x$ ; hence if  $x$  increase in arithmetic progression,  $r - x$ , and consequently hyp. log.  $v$ , will decrease in arithmetic progression, and therefore  $v$  will decrease in geometrical progression, consequently the log. of the density decreases as the altitude increases.

From experiments on the density of the air at the bottom and top of hills, Mr. CORES (Hydrostatics, p. 103.) collected, that at the altitude of 7 miles the density was four times less than at the earth's surface, or  $= \frac{1}{4}$ ; hence if  $z =$  the distance in miles above the earth's surface,  $7 : \log.$

$$\frac{1}{4} :: x : \log. v = \frac{x}{7} \times \log. \frac{1}{4}, \text{ therefore } v = \left[ \frac{1}{4} \right]^{\frac{x}{7}}. \text{ Or to express it in terms}$$

of

of the rarity  $r$ , we have  $7 : \log. 4 :: x : \log. r$ , and hence  $r = 4^{\frac{x}{7}}$ ; also

$$x = 7 \times \frac{\log. r}{\log. 4} = 1,1626 \times \log. r \text{ miles.}$$

This rule supposes the density to be as the compressive force, which is not true unless the temperature remains the same; but as the temperature is found to be very different at the same time at different altitudes, the rule will require a correction according to the altitudes of the thermometers at the two places. Omitting however this correction, the density of the air at the altitude of 45 miles is found to be 7420 times less than at the earth's surface; and yet, from observations on the twilight, the rays of light are sensibly affected by the air at that altitude.

#### ON THE BAROMETER.

53. If a glass globe be exhausted of air and balanced at one end of a beam, upon admitting the air the globe preponderates.

This experiment clearly proves that the air has weight; and from the weight necessary to balance the globe after the admission, the weight of the air will be known. Mr. COTES found the density between 800 and 900, but nearer to 900, times less than water; and Mr. HAUKEBEE made it 885 times less, when the barometer stood at  $29\frac{1}{2}$  inches. Hence, as a cubic inch of water weighs 253,18 grains troy, a cubic inch of air weighs 0,286 grains. If we take mercury to be 14 times heavier than water, the specific gravity of air : that of mercury :: 1 : 8851  $\times 14 = 12390$ .

54. If a glass tube more than 31 inches long, hemetically sealed at one end, be filled with mercury and then inverted and its end immersed in a basin of the same fluid, it will stand at an altitude above the surface of the mercury in the basin between 28 and 31 inches.

As the mercury descends from the top of the tube it must leave a vacuum, and it remains suspended by the pressure of the air upon the surface of the mercury in the basin, for if the air be taken off from the surface the mercury descends. This pressure of the air was discovered by GALILEO. He found by experiment that water might be raised by the common pump to a certain height, and no higher; whereas, had nature abhorred a vacuum, as the philosophers then thought, it might have been raised to any height. He conjectured therefore that it was owing to the air's gravitation. Afterwards his pupil TORRI-

CELLIUS



CELLIUS considered, that if the pressure of the air would support a column of water about 35 feet high, it must suspend a column of mercury, whose density is about 14 times greater, about one fourteenth part of 35 feet; he accordingly tried the experiment in the proposition and found that the mercury stood at the height which he expected. Thus he fully proved the air's pressure, and hence this is called the *Torricellian* experiment, and the vacuum which is left above is called the *Torricellian* vacuum. A tube thus filled and graduated from 28 to 31 inches is called a *Barometer*.

Hence we get the altitude of an homogeneous atmosphere; for by the last article when the mercury stood at an altitude of  $29\frac{1}{2}$  inches, the density of the air was to that of mercury :: 1 : 12390; hence the altitude of an homogeneous atmosphere =  $12390 \times 29\frac{1}{2} = 365505$  in. = 5,77 miles. If, according to some experiments, we suppose that when the mercury in the barometer stands at 30 inches the density of the air is 850 times less than that of water, the altitude of an homogeneous atmosphere would be 5,6 miles.

55. If a barometer be placed under the receiver of an air pump, and the air be exhausted, the mercury will descend; and upon admitting the air it ascends again to the former height.

56. If a bottle be partly filled with mercury, and through the cork, made air tight, a glass tube open at both ends be put so that the lower end be immersed in the mercury, then if the whole be put under the receiver of an air pump, and the air be exhausted, the mercury will rise in the tube nearly to the height at which the barometer stands at that time, or to the height at which the mercury rises in the gage.

This arises from the elasticity of the air being as its compressive force; a very small quantity therefore of air by its elasticity produces the same effect as the weight of the atmosphere. - The mercury does not rise exactly to the height, first, because you cannot exhaust all the air, and secondly, because as the mercury rises in the tube, the air in the bottle occupies a greater space, and therefore its density and elastic force is diminished and become less than that of the air in its natural state.

57. If

57. If a barometer having its lower end immersed in a basin of mercury be suspended from the beam of a balance, it is found to weigh as much as when you invert it with the same quantity of mercury in it, and suspend it by the other end.

It might at first be thought, that in the first position the weight required to balance the barometer would be only equal to the weight of the glass tube, the mercury within being supported by the pressure of the air upon the mercury without the tube; but as there is a vacuum left at the top, there is nothing to counterbalance the pressure of the air against the top on the outside; the tube therefore has to support a column of air having the same base as the base of the tube, which column of air is equal in weight to the mercury within the tube.

58. If a barometer be carried to an altitude of 54 feet, the mercury is observed to sink about  $\frac{1}{2}$  of an inch.

59. If  $a$  be the altitude of mercury in a barometer at the bottom of a mountain, and  $b$  the altitude at the top, then the altitude of the mountain

$$= 1,1626 \times \log. \frac{a}{b} \text{ miles.}$$

For at the top of the mountain the density of the air must be  $\frac{b}{a}$ , that at the bottom being unity, or we may call the rarity  $\frac{a}{b}$ , which substitute for  $r$  in the note to prop. 52. and the altitude  $= 1,1626 \times \log. \frac{a}{b}$ . The difference of temperature is not here considered.

60. When the mercury stands in the barometer at the altitude of 30 inches, the pressure of the air upon every square inch is about  $18\frac{1}{2}$  lb. troy, or a little more than 15 lb. avoirdupoise.

For a cubic inch of mercury weighs 3544 grains troy, therefore 30 cubic inches weigh about  $18\frac{1}{2}$  lb. Hence if we take the surface of a middle size man to be  $14\frac{1}{2}$  square feet, when the air is lightest the pressure

sure on him is  $13\frac{1}{3}$  tons, and when heaviest  $14\frac{3}{10}$  tons, the difference of which is 2464 lb. This difference of pressure must greatly affect us in regard to the animal functions, and consequently in respect to our health, more especially when the change takes place in a short time. The pressure of air upon the whole surface of the earth = 1204346880000000000 lb.

61. If the tube of a barometer, being perfectly cylindrical, be partly filled with mercury before it is inverted, after inversion the mercury will sink below the standard altitude, and the standard altitude will be to the depression below that altitude as the space occupied by the air after inversion to the space occupied before.

It is here supposed that the elastic force is as the density, and as the elastic force is equal to the compressive force, they balancing each other, the compressive force is as the density. If therefore the truth of this prop. appear from experiment, it follows that the elastic and consequently compressive force of the air must be as its density.

## ON THE AIR PUMP.

62. If the capacity of the barrel of an air pump : the capacity of the receiver ::  $b : r$ , after every turn, the quantity of air extracted : the quantity before ::  $b : b+r$ .

Cor. 1. Hence the quantities taken away at any number of successive turns form a geometric series, consequently the whole can never be exhausted.

Cor. 2. Hence, dividendo, the quantity remaining after every turn : the quantity before ::  $r : b+r$ , consequently the quantities which remain after any number of successive turns will form a geometric series.

63. After every turn, the density of the air is diminished in the ratio of  $b+r : r$ ; and hence after  $t$  turns, it is diminished in the ratio of  $\overline{b+r}^t : r^t$ .

64. The defects of the mercury in the gage from the standard altitude, after any number of successive



five turns, form a geometric series whose terms are in the ratio of  $b+r : r$ .

65. The altitudes of the mercury in the gage at the same time form a geometric series, the ratio of whose terms is  $b+r : b$ .

66. When the air is rarified  $n$  times, the number of turns  $= \frac{\log. n}{\log. b+r - \log. r}$ .

67. Air is necessary for the production of sound.

68. Air is necessary for the propagation of sound.

69. A candle will not burn in vacuo.

70. If the pressure of the air be taken off from one side of a thin glass phial, the pressure of the air on the other side will break it.

The experiments with the air pump, showing the very extraordinary effects of the pressure of the air, are so numerous, that it would take up too much room to insert them all here.

#### ON THE CONDENSER.

71. After every descent of the piston, one barrel of air in its natural state is forced into the receiver.

72. If the capacity of the receiver : the capacity of the barrel as  $r : b$ , then after  $t$  descents of the piston, the density is increased in the ratio of  $b : b+rt$ .

73. After any number of successive descents, the density is increased in arithmetic progression.

74. If

74. If the gage tube lie horizontal, the spaces which the air occupies after any number of successive descents of the piston will decrease in musical progression.

75. A bell in condensed air sounds louder than in air in its natural state.

The effect of condensed air may be shown by a variety of experiments. Fire engines, air guns, artificial fountains, &c. act from this cause.

#### ON PUMPS AND SYPHONS.

76. Water in pumps is raised by the pressure of the air upon the surface without.

A *common pump* is formed with two suckers, each valve of which opens upwards; the lower sucker is fixed, and the upper moveable by the handle.

The lower sucker must not be more than 36 feet above the surface of the water in the well. For the water rises by the pressure of the air upon the water without, in consequence of a vacuum within; and in the most condensed state of the atmosphere, the pressure of the air is not greater than that of a column of water 36 feet high, having equal bases. But if a cistern there receive the water, and in like manner a pump works in that, water may be raised to any height.

A *forcing pump* has two suckers, the upper of which is moveable without a valve, and the lower is fixed with a valve opening upwards.

Some forcing pumps act by the force of the upper sucker upon the water, and some by condensed air upon it.

77. If one leg of a syphon be put into a vessel of water and the air be drawn out of the other leg, the water will flow out of the leg without, provided the end be lower than the surface of the water in the vessel.

The water will continue to run till the surface of the fluid is level with the end of the syphon without, and then it will stop.

The water is made to flow through the syphon by the pressure of the air upon the surface of the water in the vessel, the air being drawn from the surface of the water in the syphon. When the legs become equal, the pressure of the air against the water at the end of the syphon without being equal to the pressure of the air on the surface of the

water in the vessel, and these having columns of water of the same altitude to support must balance each other, and consequently the fluid will then cease to flow; whereas it flowed before from the superior pressure of the column without.

78. If the syphon be capillary, the water will not flow out, till the end without be further below the surface of the fluid in the vessel than the height to which the fluid would rise in the tube by capillary attraction.

#### ON THE THERMOMETER.

79. Fluids expand by being heated, and contract again as they grow cold.

Hence a fluid whose expansion by heat is very sensible and uniform, and not subject to be frozen, is proper for the construction of a thermometer.

80. The expansion of mercury, linseed oil and spirits of wine, is, as to sense, proportional to the heat applied.

For let a thermometer constructed with these fluids be put into cold water, and then into water heated to any degree, and note the altitudes; put equal quantities of these two waters together, which will give a mean heat, and the fluid will stand at the mean altitude between the two before observed. This is found to be true, of whatever temperatures the two parts of water are. Mercury is the most proper of the three fluids, as it is capable of enduring the greatest degree of heat or cold without boiling or congelation. The thermometer usually consists of a small glass cylinder with a glass ball at the bottom, generally a globe, but it is better to make it flat, because all the mercury in it will then be the soonest affected by the variation of the air's temperature. The points where the fluid stands in the stem at *freezing* and in *boiling* water are usually noted by observation, and then the whole scale is divided into equal parts and numbered. In FAHRENHEIT's thermometer, the freezing point is at 32, and boiling water at 212. According to this division, mercury boils at 600, and blood heat is 98.

To fill a thermometer, heat the bulb and you will expel the air, then dip the other end into the fluid and it will immediately rise and fill the bulb and part of the tube; and if there be any air bubbles, whirl it round about the upper end of the stem, and the centrifugal force of the fluid being greater than that of the air, the fluid will recede from  
the



the center and drive out the air. Then heat the bulb and force the fluid to the top of the stem and hermetically seal it; and as the heat decreases, the fluid will fall and leave a vacuum above.

The pressure of the atmosphere against the outside of the bulb, not being counteracted by any air within, affects its magnitude, diminishing it as the pressure is increased. The variation however which this causes on the scale is never above one tenth of a degree.

81. If a piece of iron be heated and then left to be cooled by a current of air passing over it, in equal times, quantities of heat will be lost in proportion to the whole quantity.

When the decrements of quantities vary as the quantities themselves, these quantities must be in geometric progression. Hence the heats retained, after equal intervals of time, are in geometric progression. Sir I. NEWTON therefore heated a piece of iron red hot, and leaving it to cool, he laid upon it different metals and other fusible bodies, and noted the times when by cooling they lost their fluidity and began to coagulate; and lastly, when the heat of the iron became equal to the external heat of the human body. Thus he extended the scale to all degrees of heat. See COTES's and PARKINSON's Hydrostatics.

#### ON THE HYGROMETER.

82. Wood expands by moisture and contracts by dryness; on the contrary, chord, catgut, &c. contract by moisture and lengthen by dryness.

Hence by observing the expansion and contraction of these substances, they will show the different states of the air in respect to moisture. Various mechanical contrivances have been invented, to render sensible the smallest variations of the lengths of these substances. The twisted beard of a wild oat, with a small index fixed to it, moveable against a scale, makes a very good hygrometer; for the twisting being affected by the variation of the moisture of the air, it causes the index to move.

83. That substance is most proper for an hygrometer, whose expansion or contraction varies most nearly in proportion to the quantity of moisture imbibed.

Mr. DE LUC has made a great many experiments in order to find out those substances whose expansion increases most nearly in proportion to the quantity of moisture imbibed. The result was, that whalebone

bone and box, cut a cross the fibres, increased very nearly in proportion to the quantity of moisture, and more nearly so than any other substances which he tried. This he found by taking a quantity of shavings of each substance, and weighing them at the time when he measured the increase of the length of a slip of each, cut as above described. He however preferred the whalebone, first, on account of its steadiness, in always coming to the same point at extreme moisture; secondly, on account of its greater expansion, it increasing in length above one eighth of itself from extreme dryness to extreme moisture; lastly, it is more easily made thin and narrow. He accordingly has constructed an hygrometer with whalebone, as the most accurate for the measure of the moisture of the air. It is a little extraordinary that when he took threads of some substances in the direction of the fibres, they first increased as the quantity of moisture increased, and afterwards upon a further increase of moisture they decreased in length. See the *Phil. Transf.* for 1791.

## ON THE PYROMETER.

## 84. All metallic bodies are expanded by heat.

Various instruments have been invented to render sensible very small expansions. If the rod to be expanded act very near to an axis of motion, by a proper combination of wheels to multiply velocity, the least expansion will be perceived and may be measured.

85. Rub a piece of metal with a cloth, and the warmth which it produces in the metal will sensibly increase its length.

86. If a lamp be put under a piece of metal, the metal will gradually increase in length as it grows hotter.

In this manner Mr. MUSCHENBROEK made experiments to determine the proportion of the expansion of different metals, by applying a different number of lamps, and found the results as follows;

| Lamps. | Iron. | Steel. | Copper. | Brass. | Tin. | Lead. |
|--------|-------|--------|---------|--------|------|-------|
| 1      | 80    | 85     | 89      | 110    | 153  | 155   |
| 2      | 117   | 123    | 115     | 220    | *    | 274   |
| 3      | 142   | 168    | 193     | 275    | *    | *     |
| 4      | 211   | 270    | 270     | 361    | *    | *     |
| 5      | 230   | 310    | 310     | 377    | *    | *     |

Tin melted with two lamps and lead with three. With this kind of pyrometer Mr. FERGUSON found the expansion of metals to be in the following proportion; iron and steel 3, copper  $4\frac{1}{2}$ , brass 5, tin 6, lead 7. An iron rod 3 feet long is about one 70th of an inch longer in summer than in winter.

87. If

87. If a metal be put into water and the water be heated, the metal expands as the water increases in heat.

By this method Mr. SMEATON determined the expansion of different metals, for by means of a mercurial thermometer immersed in the water he could always ascertain the degree of heat. He found that in equal intervals of time the expansions were in geometric progression. By this he was enabled to get the measure of the bar before it was applied to the instrument. This will be best understood by explaining an experiment. The time elapsed between applying the bar to the instrument and taking the first measure, was  $\frac{1}{2}$  a minute; therefore the intervals between taking the succeeding measures were  $\frac{1}{2}$  a minute also. The first measure was 208; the second 214,5; the third 216,5; the fourth 217,5. The differences of these are 6, 5; 2; 1. Now these three numbers are nearly equal to 6, 3; 2, 25; 0, 8, which from a geometrical progression whose common ratio is 2, 8. As therefore we may suppose the expansion from the instant the bar was applied to the time of taking the first measure followed the same law, we can find the expansion in the first  $\frac{1}{2}$  minute (at the end of which the first measure was taken) by continuing back the progression, or multiplying 6, 3 by 2, 8, which gives 17,7 for the lengthening the first  $\frac{1}{2}$  minute; hence  $208 - 17,7 = 190,3$  for the measure before the bar was applied. The following expansions are selected from Mr. SMEATON's table, showing how much a foot in length of each increases in decimals of an inch by an increase of heat corresponding to 180 degrees of FAHRENHEIT's thermometer, from freezing to boiling water. See Mr. SMEATON's account in the *Phil. Trans.* 1754.

|                            |             |            |       |       |
|----------------------------|-------------|------------|-------|-------|
| White glass barometer tube | ,01         | Cast brass | - - - | ,0225 |
| Hard steel                 | - - - ,0147 | Grain tin  | - - - | ,0298 |
| Iron                       | - - - ,0151 | Lead       | - - - | ,0344 |
| Copper hammered            | - ,0204     | Zinc       | - - - | ,0353 |

Metals being thus subject to expansion by heat, a pendulum made with a single rod of metal will continually be subject to a variation in its length from the variation of the temperature of the air. To correct this Mr. HARRISON invented a pendulum, called a gridiron pendulum, composed of rods of iron and rods of brass, so connected together, that the brass expands upwards when the iron expands downwards; by this means the distance from the point of suspension to the center of oscillation is subject but to a very small variation. Mr. GRAHAM invented the following method of preserving the length of the pendulum the same in different temperatures. He took a glass, or metallick tube, and put in some mercury; now the heat, by expanding the glass or metal downwards, expanded the mercury upwards; by the adjustment therefore of a proper quantity of mercury, he could make these effects in altering the length of the pendulum nearly destroy each other. He found the errors of a clock of this sort to be but about  $\frac{1}{8}$  of the errors of the best clock of a common sort.



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# O P T I C S.

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## D E F I N I T I O N S.

1. **W**HATEVER grants a passage to light is called a medium.  
2. By rays of light is understood its least parts, either successive in the same lines, or cotemporary in several lines.

It is clear that light consists of parts both successive and cotemporary, because in the same place you may stop that which comes one moment, and let pass that which comes immediately after. The least sensible part which may be stopped, or suffered to proceed, is called a ray of light.

3. *Refrangibility* is that disposition of a ray of light to be refracted, or turned out of its course, when it passes out of one medium into another.

When a ray of light passes out of a rarer medium into a denser, Sir I. NEWTON supposes that it is refracted by the superior attraction of the denser medium, and by that means drawn out of its course.

4. *Reflexibility* is that disposition of a ray of light to be reflected, or turned back into the same medium from any other medium upon whose surface it may fall.

Sir I. NEWTON supposes that light is not reflected by impinging upon the solid parts of the body, but by some power of the body which is evenly diffused all over its surface, and by which it acts upon the ray and impels it back without immediate contact.

5. *Inflection* is that disposition of a ray of light to be turned out of its course when it passes very near to the edges of bodies.

6. The angle of incidence is the angle which the line described by the incident ray makes with the perpendicular to the reflecting or refracting surface at the point of incidence.

7. The angle of reflection or refraction is the angle which the line described by the reflected or refracted ray makes with the perpendicular to the reflecting or refracting surface at the point of incidence.

8. Any parcel of rays diverging from a point, considered as separate from the rest, is called a *pencil* of rays.

9. A lens is a medium bounded by two spherical, or one plain and one spherical surface; and the line joining the centers, or which passes perpendicularly through each surface, is called the *axis*.

There are 6 lenses, a double convex, a double concave, a plano-convex, a plano-concave, a concavo-convex and a meniscus.

10. The focus of rays is that point from which they diverge, or to which they converge.

11. The focus of parallel rays is called the *principal focus*.

## ON THE GENERAL PROPERTIES OF LIGHT.

1. If the sun's rays be admitted into a dark room perpendicularly through a circular aperture, they form a cone of bright light, decreasing till it comes to the vertex, where the rays cross, and it then increases; about this there is a kind of penumbra, or fainter light, which is terminated by lines drawn from the sun to the opposite sides of the aperture.

Hence the image of the sun received within the room is a bright central light surrounded with a fainter light; and if  $r$  = the radius of the aperture,  $t$  = the tangent of  $16'.2''$ , the mean apparent semidiameter of the sun; then the radius of the whole image at the distance  $x$  from the aperture =  $r + tx$ ; also the radius of the bright central part =  $r - tx$ , or  $tx - r$  according as you take it before or after the intersection. Hence when the radius of the aperture becomes evanescent the penumbra vanishes.

2. The image of an aperture of any figure will approach towards a circle as you receive it further from the aperture.

For the diameter  $2r + 2tx$  of the image approaches to  $2tx$  as its limit, by increasing  $x$ ; therefore however irregular the figure of the aperture may be, all the diameters of the image will approach to a ratio of equality, and consequently the image will approach to a circle as its limit.

3. Lights which differ in colour have different degrees of refrangibility.

4. The sun's light consists of rays of different colours and differently refrangible.

If the sun's rays be admitted into a dark room through a small hole in a window shutter, and be refracted through a prism, the image is not round, but a long figure with parallel sides and semicircular ends, the length of which is about five times its breadth; that end which has suffered the least refraction is red, and that which has suffered the greatest is violet. The whole image consists of seven distinct colours, lying in the following order, red, orange, yellow, green, blue, indigo, violet;

violet; the red is the least refrangible, and the other more in their order. These are called primary colours, all other colours being only different combinations of these. Each colour forms a distinct image of the sun, which images, in this experiment, running into each other, make a gradual change of colour in the image. But if a convex lens be placed before the prism, each image will be diminished, and by that means they will be separated and each rendered distinct.

If two coloured images be formed with two prisms, and thrown one upon the other, then if that image be looked at through a prism, the images will be again separated.

5. The primary colours cannot be separated into other colours by any refraction.

For if in the last experiment all the colours but one be stopped, for instance, the red, and that be again refracted by a prism, it suffers no alteration in colour. By suffering the colours to pass in succession, from the red, each preserves its colour, but the quantity of refraction keeps increasing. The image of each colour is perfectly circular, which shows that the light of each colour is refracted regularly without any dilatation of the rays; it is therefore uncompounded, or homogeneous.

6. If the breadth of each colour in the spectrum formed by the prism be measured, it will appear that the breadth of the red, orange, yellow, green, blue, indigo, violet, are as the numbers 45, 27, 48, 60, 60, 40, 80, respectively.

If the circumference of a circle be divided into  $45^\circ$ ,  $27^\circ$ ,  $48^\circ$ ,  $60^\circ$ ,  $60^\circ$ ,  $40^\circ$ ,  $80^\circ$ , and the respective sectors be painted red, orange, yellow, green, blue, indigo, violet, and the circle be turned swiftly, it will appear nearly white. For the ideas we have from the impression of light remain for a short time, and thus the colours excite the same sensation as if they all entered the eye collected together.

7. If the direct image of the sun through a small hole be received upon a screen perpendicular to the rays, and the rays be then intercepted by a prism and fall perpendicularly on the first side, if the distance from the place of the direct image to the nearest edge of the red and farthest of the violet be measured, they will be the tangents of the angles of  
of



of deviation, the radius of which is the distance from the point where the rays emerge to the place of the direct image.

The angle of incidence on the second side of the prism  $\pm$  the refracting angle of the prism, to which add the deviations of the two extreme colours, and we get the two angles of refraction, the sines of which will be to the sine of incidence as 77 and 78 to 50. Hence if the difference between 77 and 78 be divided in the ratio of the breadth of each colour, it gives 77,  $77\frac{1}{8}$ ,  $77\frac{1}{3}$ ,  $77\frac{1}{2}$ ,  $77\frac{2}{3}$ ,  $77\frac{5}{6}$ , 78 for the sines of refraction, the common sine of incidence being 50; that is, the sine of incidence : the sine of refraction of the red rays :: 50 : not less than 77 nor greater than  $77\frac{1}{8}$ , the boundary of the red; and the same for the rest.

8. Candle light is of the same nature as the light from the sun.

For rays from a candle may be separated into all the different colours, and they lie in the same order as in the light from the sun.

9. The sun's light consists of rays which differ in reflexibility, and those rays which are most refrangible are most reflexible.

For after forming a coloured image, as before, with a prism, by turning the prism about its axis, until the rays within it, which in going out into the air were refracted at its base, become so oblique to the base as to begin to be totally reflected thereby, those rays become first reflected, which before at equal incidences with the rest had suffered the greatest refraction.

10. According to Sir I. NEWTON, the colours of natural bodies arise from hence, that some reflect one sort of rays and others another sort more copiously than the rest.

For every body looks most splendid in the light of its own colour, and therefore it reflects that the most copiously. Besides, by reflection you cannot change the colour of any sort of rays; and as bodies are seen by reflection, they must appear of the colour of those rays which they reflect. This is the opinion of Sir I. NEWTON. But Mr. DELAVAL accounts for the colours of natural bodies in a manner different from this. See the *Manchester Memoirs*, Vol. II.

11. Thin transparent substances, as glass, water, air, &c. exhibit various colours according to their thickness.

For a very thin glass bubble, or a bubble of water, will appear to have concentric colours; the bubble blown with water, first made tenacious by dissolving a little soap in it, continually grows thinner at the top by the subsiding of the water, the rings of colours dilating slowly, and overspreading the whole bubble. A convex and concave lens of nearly the same curvature being pressed closely together, exhibit rings of colours about the point where they touch. Between the colours there are dark rings, and when the glasses are very much compressed, the central spot is dark. Sir I. NEWTON, to whom we owe all these discoveries, found the thickness of the air between the glasses where the colours appeared to be as 1, 3, 5, 7, 9, &c. and the thickness where the dark rings appeared to be as 0, 2, 4, 6, 8, &c. The coloured rings must have appeared from the reflection of the light, and the dark rings from the transmission of the light. The rays therefore were transmitted when the thickness of the air was 0, 2, 4, 6, 8, &c. and reflected at the thicknesses 1, 3, 5, 7, 9, &c. Sir I. NEWTON therefore supposes, that every ray of light in its passage through any refracting surface is put into a certain constitution or state, which in the progress of the ray returns at equal intervals, and disposes the ray at every return to be easily transmitted through the next refracting surface, and between the returns to be easily reflected by it. These he calls fits of easy transmission and reflection.

12. If a beam of the sun's light be let into a dark room, the shadow of an opaque body is larger than it ought to be, upon supposition that the rays of light proceeded by it in straight lines.

13. If the edges of two knives be placed parallel to each other at the distance of about the 400th part of an inch, and a ray of light fall upon them and some part pass between their edges, the stream of light will part in the middle at the knives and leave a shadow between the two parts.

Hence it appears, that bodies act upon light at a distance; and in the case of the last proposition it appears, that the body acted upon the light at the distance of the 800th part of an inch. In the former case the light was bent from the body, and in the latter towards it. This is called the *inflection* of a ray of light.

14. When

14. When a ray of light moves in a medium denser than its ambient medium, and comes to its surface and is reflected, the reflection will be stronger the rarer the ambient medium is; and the total reflection will take place at a greater angle of incidence the less the difference of the densities of the medium is.

For if the two mediums had the same density, it would be the same as the continuation of the same medium, in which case no reflection would take place; the reflection therefore will be the stronger by how much rarer the ambient medium is. Also, by prop. 31. the total reflection takes place after the sine of refraction becomes radius; now the sine of incidence : the sine of refraction, nearer to a ratio of equality the denser the ambient medium is, and therefore the sine of incidence must approach nearer to radius, as the limit, when the total reflection takes place, and consequently the angle of incidence becomes greater, the denser the ambient medium is, or the less the difference of the densities is. The refractive power is here supposed to vary as the density.

Light therefore is not reflected by striking upon the solid parts of the surface of that body upon which it is incident, for if it were the denser ambient medium would cause the strongest reflection. It is therefore reflected by some power diffused over the surface.

15. The forces of bodies to reflect and refract light are nearly as their densities, except that unctuous and sulphureous bodies refract more than others of the same density.

For oil olive, spirit of turpentine and amber, which are sulphureous unctuous substances, have their refractive powers nearly as their densities; but their refractive powers are two or three times greater in respect to their densities than the refractive powers of bodies which are not sulphureous or unctuous.

#### ON THE FOCI OF, AND IMAGES BY, REFLECTED RAYS.

16. The angle of incidence is equal to the angle of reflection, and the plane passing through the incident and reflected rays is perpendicular to the surface.

17. Parallel



## 17. Parallel rays are reflected parallel.

In fact no pencil of rays can be accurately parallel; yet if the body from which the rays diverge be not nearer than a mile, all the rays in any pencil which we have ever any occasion to consider are, as to sense, parallel.

18. Diverging rays reflected at a plane surface, after reflection will diverge from a point at the same distance on the other side, and in the same perpendicular.

19. If parallel rays fall very nearly perpendicularly on the concave side of a spherical reflector, after reflection they will all converge, very nearly, to the middle of that radius to which they are parallel: if they fall on the convex side, they will diverge from that point.

Hence the middle of the radius is the principal focus; it is also called the geometrical focus.

In optics, so far as regards practical purposes, we have occasion only to investigate the focus of rays falling very nearly perpendicularly upon the reflecting or refracting surfaces; for in practice the breadth of that surface is very small in respect to the radius, and the rays fall nearly perpendicularly; the rules therefore here given, are sufficiently accurate for the practical optician.

20. If the focus of diverging or converging rays, falling very nearly perpendicularly upon a spherical surface, and the center be joined, the distance of the principal focus from the focus of incident rays : the distance of the principal focus from the center or surface :: that distance : the distance of the principal focus from the focus of reflected rays upon the same line.

Cor. 1. Hence the distances of the foci of incident and reflected rays from the center are in the same ratio as their distances from the surface.

Cor. 2. Hence also if  $d$  = the distance of the focus of incident rays from the surface,  $r$  = the radius of the surface, the distance of the focus of reflected rays from the surface =  $\frac{dr}{2d \oslash r}$ .

21. The

21. The focus of reflected rays in the last prop. lies the same way from the principal focus as the focus of incident rays, but on different sides of either the center or surface.

The two foci always move in opposite directions, and coincide at the center and surface.

22. If the focus of incident rays move, its velocity : the velocity of the reflected rays :: the square of the distance of the principal focus from the focus of incident rays : the square of half the radius.

23. If parallel rays be reflected at a spherical surface, the sine of half whose arc  $= s$ , and radius  $= r$ ; then when  $s$  is small in respect to  $r$ , the longitudinal aberration from the geometrical focus  $=$

$$\frac{s^2}{4r} \text{ nearly; and the lateral aberration} = \frac{s^3}{2r}.$$

When the rays in a pencil diverge from a point, and either by reflection or refraction are brought all together again, they then form a luminous point corresponding to that from which they diverged. By this means a new visible object is formed, called the image of the other; for the eye now receives the rays as coming from this latter point, and therefore it judges the former point to be in the place of the latter. And as this is true for every point of any object, every object may thus actually be formed anew, so far as regards our visible ideas. And the rays diverging to the eye from the image thus formed, after the same manner as if they came directly from the object, excite an idea of that image, or of an object equal and similar to it. Now if the pencils of rays which diverge from all the points of an object be again respectively collected at the same distances, they then form a new visible object equal to that from whence they flowed; but if the points of this new object, called the image, corresponding to those of the original object, be at a greater or less distance, they then form a new visible object greater or less than the original one. Thus therefore we are able to form a new visible object, very near to us, exactly similar to an object at a great distance. I call this a visible object, because at the place where it is formed there are no corresponding tangible ideas, as in the object from whence the rays first flowed; but in respect to our visible ideas, which we are here only considering, it is as much an object

object as the other. The eye therefore may be so situated in respect to this new object, that it may appear much greater than the original object, every object appearing greater the nearer it is to the eye. Now in respect to the brightness of this new visible object, we may consider, that when the eye looks directly at any object, it receives no more rays from any one point than what can enter the pupil; but when an image is formed by a lens, for instance, all the rays from any one point of the object which fall upon the lens are collected together and form a point of the image. Now if the diameter of the pupil of the eye = 0,1 in. and the diameter of the lens = 5 inches, their areas will be as 0,01 : 25, or as 1 : 2500; there are therefore, *ceteris paribus*, 2500 times as many rays collected together to form every point of the image by the lens as enter the eye and form the image, supposing all the rays to be refracted. Now although the rays diverge from every point of this image formed by the lens, and therefore where the eye is situated it may not receive them all, yet it being situated near to it, it will receive a very considerable part, and the more the nearer it is. Hence the number of rays which the eye receives from any point of this image may be greater than that which it receives directly from the object, and thus the image may be brighter than the object. These are the reasons why any distant object may be made to appear larger and brighter. And the common expression, that the object is brought nearer, is not incorrect; for the visible object is actually nearer; but it not being accompanied with any tangible ideas, we call it an image of the other; whereas it is a visible object formed by the same rays as the original visible object was. Looking therefore at the visible object thus formed, we get an idea of the original visible object seen under the same angle, and from thence, by association, we conclude what are the corresponding tangible ideas.

24. If any object be placed before a plane reflecting surface, the image will be equal to it, and situated at the same distance on the other side.

Cor. Hence if a man look at his own image in a plane reflector, it appears at the same distance on the other side, and is equal in magnitude to himself. Also if he view the whole of his image in a plane reflector, he appears to fill a space in the reflector equal to half his length and half his breadth, or one fourth of his area considered as a plane figure.

The best reflecting surfaces which we have are supposed to reflect not above one half the light which falls upon them; the rest enters and is lost.

25. If two plane speculums be placed parallel to each other, and any object be put between them, a number of images will appear in each speculum  
situated



situated one behind another in a perpendicular to the speculums passing through the object.

If  $d$  be the distance of the speculums, and  $x$  the distance of the object from one of them; then the distances of the images seen in that speculum from that speculum will be  $x, 2d-x, 2d+x, 4d-x, 4d+x, \&c.$  and in the other speculum the distances of its images from it will be  $d-x, d+x, 3d-x, 3d+x, \&c.$  The further the images are from the speculum the fainter they are.

26. If the speculums be inclined to each other, a set of images will appear in the circumference of a circle whose radius is the distance of the object from the concourse of the planes.

## L E M M A.

If  $a$  be an arc of a circle of  $1^\circ$  to the radius 1, and  $m$  be to  $n$  as any other arc is to its radius; then  $\frac{a}{1} : \frac{m}{n} :: 1^\circ : \frac{am^\circ}{n}$  the angle subtended by  $m$ .

27. If an object be spherical and concentric with a spherical reflector, the image will be spherical and concentric also with it.

Cor. 1. If the object be a line, the magnitudes of the object and image are as their distances from the center or surface, that is, as  $d : \frac{dr}{2d \oslash r}$ . If the object whose magnitude is  $m$  be situated between the principal focus and surface, and an eye be situated at a greater distance

$D$  from the surface, then, by the lemma,  $\frac{am^\circ}{D-d}$  and  $\frac{amr^\circ}{D \times r - 2d + dr}$  will be the apparent angles under which the object and image appear. Hence when  $d = \frac{1}{2}r$ , or the object be situated in the principal focus, the apparent magnitude of the image  $= \frac{2am^\circ}{r}$ , which is the same where-

ever the eye is placed. If the object be a spherical surface, we must take the duplicate ratio of  $d : \frac{dr}{2d \oslash r}$  for the ratio of their magnitudes.

The angles are here supposed to be small. *See p. 163.*

Cor. 2. The object and image coincide, and consequently become equal at the center and surface.

Strictly speaking, when the object is at the surface it cannot be reflected to form an image, and at the center the object can be only a

M

point;

point; the corollary therefore must be thus understood, that as the object approaches the center or surface, the image approaches it at the same time, and the distance between them keeps diminishing sine limite.

28. The image is erect when it is on the same side of the center with the object, and inverted when on the contrary side.

When the object is beyond the center in respect to the surface, the image is between the center and principal focus, by prop. 21. and therefore it is inverted; but if the eye should receive the rays reflected from the speculum before the image is formed, the image would appear erect; then as the eye recedes from the speculum, the image will grow confused, and when the eye gets to the place of the image after reflection, nothing distinct can then be seen, for the eye is then looking at an image close to itself, and therefore there must be the same confusion as when the eye looks at an object close up to it; after that, as the eye recedes further back, the image will then begin to appear inverted, because the eye will then look at an inverted image. And in general, although the reflector or lens may form an inverted image, if the rays enter the eye before the formation of the image, it appears erect; but when they enter the eye after the image is formed, it appears inverted.

29. The image of a straight line by reflection at a spherical surface is a conic section.

If the object be placed at the distance of half the radius of the reflector from the center, the image will be a parabola; if further from the center, an ellipse; if nearer to the center, an hyperbola.

#### ON THE FOCI OF, AND IMAGES BY, REFRACTED RAYS.

30. When a ray of light passes out of one medium into another, the sine of incidence is to the sine of refraction in a given ratio.

We are here to understand rays of the same colour. The sines of incidence and refraction of the most and least refrangible rays out of glass into air are as 50 : 77 and 78; hence for the mean rays the ratio is 20 : 31, or nearly as 2 : 3, which is, in common, taken for the ratio of all rays. Out of rain water into air, these ratios are as 81 : 108 and 109; for the least refrangible rays therefore the ratio is 3 : 4, which, in common, is taken for that of all the rays.

31. Light cannot pass out of a denser medium into a rarer, when the sine of incidence has a *greater* proportion to radius than it has to the sine of refraction.

The limit is when the proportion is the *same*. Hence a ray cannot pass out of water into air at a greater angle of incidence than  $48^{\circ}.36'$ . the sine of which  $= \frac{3}{4}$  of radius. Out of glass into air the angle must not exceed  $40^{\circ}.11'$ . When the angle however is within the limit for the light to be refracted, some of the rays are reflected.

32. As the angle of incidence increases, the angle of deviation will increase.

33. Parallel rays of the same colour falling upon a plane refracting surface, will continue parallel after refraction.

34. Parallel rays of the same colour passing through a medium bounded by plane parallel surfaces, will continue parallel.

35. If parallel rays pass through a prism whose refracting angle is small, and the angle of incidence be also very small, and  $I$  and  $R$  be the angles of incidence and refraction,  $G$  the refracting angle, then the angle of deviation after the rays have passed through the prism  $= \frac{G \times \overline{I - R}}{R}$ ,

Cor. Hence the deviation is in proportion to the refracting angle.

36. If diverging or converging rays fall very nearly perpendicularly upon a plane refracting surface, the distance of the focus of incident rays from the surface : the distance of the focus of refracted rays :: the sine of refraction : the sine of incidence, or as  $R : I$ .



We shall use  $I : R$  to express the ratio of the sine of incidence : the sine of refraction.

37. If diverging rays pass nearly perpendicularly through a medium bounded by two plane parallel surfaces, the distance between the foci of incident and emergent rays : the distance of the surfaces ::  $I : I - R$ ,  $I$  and  $R$  being the ratio of the sines of incidence and refraction at the first surface.

38. If parallel rays fall very nearly perpendicularly on a spherical refracting surface, the distance of the focus of refracted rays from the surface : the radius of the surface ::  $I : I - R$  or  $R - I$ .

This focus is called the principal focus.

39. If diverging or converging rays fall very nearly perpendicularly upon a spherical refracting surface, the distance of the focus of incident rays from the principal focus : its distance from the center :: its distance from the surface : its distance from the focus of refracted rays.

Cor. 1. Hence the two foci coincide at the center and surface.

Cor. 2. The four distances in the proposition lie all the same way from the focus of incident rays, or two on each side.

A concave surface of a denser medium and a convex of a rarer give a divergency to rays; and a convex surface of denser and concave of a rarer medium give a convergency.

40. If parallel rays fall very nearly perpendicularly upon a sphere, and that diameter to which they are parallel be produced, if necessary; the focus after the rays have passed through the sphere will bisect the distance between the focus at the first surface and the extremity of the above diameter which is on the contrary side to the incident rays.

If

If  $r$  = the radius of the sphere, the distance of the principal focus from the center  $= \frac{rI}{2I-2R}$ . If the sphere be water, the focus lies at the distance of a radius beyond its surface; if glass, that distance is half the radius.

41. If the line joining the centers of the surfaces of a lens be divided in the ratio of the respective radii, all the rays passing through that point will have their incident and emergent parts parallel.

That point is called the *center* of the lens.

Hence if the thickness of the lens be inconsiderable, the passage of every ray which passes through the center may, for all practical purposes, be considered as a straight line.

42. If the sine of incidence out of air into a double convex or concave lens : the sine of refraction ::  $I : R$ , and the radii of the two surfaces be  $m$  and  $n$ , the distance  $D$  of the principal focus from the center of the lens  $= \frac{mn}{m+n} \times \frac{R}{I-R}$ , the thickness of the lens being inconsiderable, and the rays falling nearly perpendicularly.

Cor. 1. If the lens be glass, and  $I$  be taken to  $R :: 3 : 2$ , then  $D = \frac{2mn}{m+n}$ ; and if  $m = n$ ,  $D = m$ .

Cor. 2. If one radius  $n$  become infinite, or the lens become plano-convex or concave,  $D = m \times \frac{R}{I-R}$ . Hence for such a glass lens,  $D = 2m$ .

Cor. 3. The focal length is the same on which ever of the sides the rays fall, the radii being similarly involved. Also, all lenses of the same focal length must have the same effect.

43. If the lens be a meniscus, or concavo-convex,  $D = \frac{mn}{m-n} \times \frac{R}{I-R}$ .

These

These rules, though not mathematically true, are sufficiently accurate for all practical purposes.

Parallel rays on a convex lens, plano-convex lens and meniscus, converge to the principal focus; but on a concave lens, plano-concave and concavo-convex lens, they diverge from the principal focus. As therefore the rays would return back in the same lines, rays diverging from the principal focus in the former case, and converging to it in the latter, become parallel after passing through the lens.

To find the principal focal length of a convex lens, hold it parallel to a screen which is perpendicular to the sun's rays, and remove it backwards and forwards till you find the bright spot the least you can make it, and the distance of the lens from the screen is its focal length. Or remove it till the image be equal to the lens, and the distance is equal to twice the focal length. If the lens be concave, remove it from the screen till the bright annulus surrounding a darker central circle be equal in diameter to twice the diameter of the lens, and the distance of the lens from the screen is the focal length. The circle in the center is darker than that part without, because it receives only those rays which pass through the lens, whereas the annulus beyond receives those rays which pass through the lens, and the direct rays of the sun also. The part beyond the annulus receives the direct rays only, and therefore is darker than the annulus, but it is brighter than the central circle, because the direct rays after refraction have their density diminished by being rendered diverging. Hence therefore the quantity of light in the annulus is equal to the sum of the quantities on each side.

44. If diverging or converging rays fall nearly perpendicularly upon a lens, the distance  $d$  of the focus of incident rays from the principal focus: its distance from the center :: that distance : the distance  $D$  of the focus of incident and refracted rays.

Hence, and from prop. 42.  $d \propto \frac{mn}{m+n} \times \frac{R}{I-R} : d :: d : D =$   
 $\frac{\overline{m+n} \times \overline{I-R} \times d^2}{m+n \times I-R \times d \propto mnR}$ , for a convex lens; for a concave  $D =$   
 $\frac{\overline{m+n} \times \overline{I-R} \times d^2}{m+n \times I-R \times d + mnR}$ . Hence the distance of the focus of refracted

rays from the lens  $= \frac{mnRd}{m+n \times I-R \times d + mnR}$ . Hence the distance of the focus of incident rays from the lens : the distance of the focus of refracted rays from the lens ::  $\overline{m+n} \times \overline{I-R} \times d + mnR : mnR$ . For a glass lens, this ratio becomes  $\overline{m+n} \times \overline{d} + 2mn : 2mn$ .

The



The focus of refracted rays lies the same way from the focus of incident rays as the principal focus does.

By moving the focus of incident rays the focus of refracted rays moves in the same direction; and they meet at the lens.

The proportion in the proposition holds for rays falling on a sphere, only assuming its principal focal length from the center, instead of the principal focal length of the lens.

45. If a medium be bounded by an ellipse or hyperbole revolving about its major axis, and the major axis : the distance of the foci ::  $I : R$ , all rays parallel to the major axis, entering into the former or coming out of the latter, will be refracted to the other focus.

46. Let the sine of incidence be to the sine of refraction of the least and most refrangible rays, as  $p$  to  $m$  and  $n$  respectively, then if parallel rays fall on a plano-convex lens, the diameter of the aperture : the diameter of the circle of aberration in the focus of the lens ::  $n - m : n + m - 2p$ .

Cor. 1. Hence if the lens be glass,  $p$ ,  $m$  and  $n$  are 50, 77 and 78 respectively; hence the diameter of the circle of aberration =  $\frac{1}{3}$ th part of the diameter of the aperture.

Cor. 2. Hence also the angle of aberration varies as the diameter of the aperture directly and the focal length inversely.

47. If parallel homogeneous rays fall upon the plane side of a plano-convex lens, and  $n : m ::$  the sine of incidence out of glass into air : the sine of refraction, also let  $r =$  the radius of the aperture of the lens, and  $d =$  the distance of the principal focus from the surface of those rays which fall very near the center; then the longitudinal aberration

$$= \frac{m^2}{m - n^2} \times \frac{r^2}{2d}, \text{ and the lateral aberration} = \frac{m^2}{m - n^2} \times \frac{r^3}{2d^2}.$$

48. The

48. The diameter of the circle of aberration is equal to half the lateral aberration in the last proposition.

If the lens be glass, and we take  $n : m :: 20 : 31$ , also if  $r = 2$  inches, and the radius of the surface  $= 600$  inches, the diameter of the circle of aberration  $= \frac{31^2 \times 8}{20^2 \times 4 \times 600^2} = \frac{961}{72000000}$  of an inch. This is the

aberration from the spherical form of the glass. Now the aberration from the different refrangibility of the rays  $= \frac{4}{3}$  by prop. 46. Hence the former aberration : the latter  $:: 1 : 5449$ . The aberration therefore from the form of the glass is so small that it may be neglected. Before Sir I. NEWTON, the imperfection of refracting telescopes was supposed to arise from the spherical figures of the glasses; but he has thus shown that it arises principally from the different refrangibility of light. He proposed therefore to form the image of the object by reflection, which would not be subject to a like imperfection, and for this purpose he constructed a reflecting telescope.

As the aberration of rays from the different refrangibility of light is so great, it might be expected that an image could not be formed so free from colour as we find it; but the rays are not scattered uniformly over the circle of aberration, but are much more dense towards the middle, the density varying as the distance from the circumference directly and distance from the center inversely, nearly. On account therefore of their quick increase of rarity as you recede from the center, only those rays which are near the center are strong enough to be visible.

49. An object in the water looked at in a direction perpendicular to the surface appears elevated  $\frac{1}{4}$  of its depth in the water.

As the eye recedes from that perpendicular the object appears to rise, and would appear at the surface if the eye were removed to an infinite distance. If a river be 6 feet deep it appears to be only  $4\frac{1}{2}$  ft.; a person not apprised of this might venture into water at the hazard of his life.

50. The image of a straight line by refraction at a plane surface, is a straight line, but not equal to the object, or similarly situated, unless the object is parallel to the surface.

All plane objects will be similar to their images when they are parallel to the surface; otherwise not. Hence also a straight rod put into water appears bent at the surface, the image of the part within lying above the object.

51. An

51. An object seen through a medium bounded by two plane parallel surfaces whose distance is  $d$ ,

appears nearer by  $d \times \frac{I-R}{I}$  by prop. 37.

For the image of an object is formed by the images of every point of the object. If the medium be glass it appears nearer by  $\frac{1}{3}d$ ; therefore an object seen through common glass does not appear to be sensibly altered.

52. An object seen through a very small hole appears inverted, because the rays from the extreme parts of the object must have crossed at the hole.

The rays of light are probably inflected as they pass by the edge of the hole, for it renders distant objects very distinct to a short sighted person, and has therefore the property of a concave lens. This effect however may perhaps partly arise from the small number of rays admitted to the eye, by which the confusion to a short sighted person may in some measure be taken off. But from my own experience of the appearance of objects thus seen, I can have no doubt but that the former effect is produced.

53. The image of a spherical object concentric with a lens, will be also spherical and concentric with it.

54. The image will be erect or inverted, according as it is on the same or different sides of the center in respect to the object.

In a convex lens the image will be erect when it is further from the lens than the principal focal length, and inverted when nearer. In a concave lens the image is always erect.

When the image is inverted, if the eye receive the rays before they form the image, the object appears erect; then as the eye recedes from the lens, a confusion will come on till the eye gets to the image, where nothing will appear; afterwards the object appears inverted.

55. If  $r$  be the principal focal length,  $d$  the distance of the object from the lens, then the distance

N

tance



tance of the image from the lens  $= \frac{dr}{d \mp r}$ , where the lower or upper sign takes place according as the lens is concave or convex.

In a concave lens the image is always nearer than the object; in a convex lens it may be nearer or further off.

56. If the object be an arc of a circle, the magnitude of the object : the magnitude of the image  $:: d \mp r : r$ ; but if it be a spherical surface, their magnitudes are as  $\overline{d \mp r}^2 : r^2$ .

57. The apparent diameters of an object to the naked eye, and seen through a lens, are as their magnitudes divided by their distances from the eye.

An object always appears diminished when seen through a concave lens, and magnified in a convex lens when nearer than the principal focus, unless the lens is close to the object or eye, in which cases the apparent magnitude is not altered. If a linear object whose magnitude is  $v$  be placed beyond the principal focus of a convex lens, and the eye be on the other side at the distance  $m$  from the lens, the apparent magnitude to the naked eye, and the apparent magnitude in

the lens will, by the lemma, be  $\frac{av^0}{d+m}$  and  $\frac{av^0 r^0}{dr - d - r \times m}$ . Now if  $d=r$ ,

or the object be situated in the principal focus, these become  $\frac{av^0}{m+r}$  and

$\frac{av^0}{r}$ . Which shows that as the eye recedes from the lens, the apparent

magnitude of the object to the naked eye diminishes, and the apparent magnitude of the image will remain the same. If  $m=r$ , the magni-

tudes become  $\frac{av^0}{r+d}$  and  $\frac{av^0}{r}$ ; hence the apparent magnitude of the

image is the same at all distances of the object from the lens. In all these cases the angles are supposed to be very small. *see pa. 163.*

In vision by images, as Mr. HARRIS in his excellent Treatise of Optics observes, we are generally deprived of many circumstances by which we usually judge of distance, which makes it very difficult in most cases to judge of the place of an image, if it be further off than

2 or 3 yards. These difficulties are again increased by some peculiarities belonging to images, which we are not accustomed to observe, and for which therefore we are at a loss how to make proper allowances.

In respect to the idea of apparent magnitude, as the same author observes, it is difficult to ascertain precisely, how far, or in what proportion, apparent distance affects it. The visual angle may be the same and yet the apparent magnitude very different: a pane of glass, for instance, does not appear so big as the front of an house seen through it; nor does a child 2 feet high appear so big as a man 6 feet high at three times the distance, although in both cases the visual angle is the same. If we suppose an object to be at a greater distance, and subtend the same angle as one at a less distance, we have an idea of a greater apparent magnitude. Mr. HARRIS therefore supposes, that *apparent magnitudes are either exactly, or very nearly, in the compound ratio of the visual angles and apparent distances.* Hence when objects are so near that the apparent distances are judged to be the same as the true, the apparent magnitude is not altered by altering its distance; thus a man appears as big at the distance of 6 yards as he does at 1 yd. And it seems to be necessary, that we should have some certain means of judging of near distances, otherwise we might be frequently in great danger without perceiving it. But when, by increasing the distance of an object, or its image, the apparent distance does not increase so fast as its true distance, the apparent magnitude diminishes. For if  $m$  = the magnitude of the object, or its image,  $d$  = the real distance from the eye,  $D$  = the estimated distance; then the apparent magnitude =  $\frac{m}{d} \times D$ ; hence, as long as  $D = d$ , the apparent magnitude is the same at all distances; but when  $D$  increases slower than  $d$ , which is the case when they become of considerable magnitude, then the apparent magnitude diminishes. Hence, as we are led to judge of the situation of an object, and more particularly so of an image, to be so very different from their true place, it happens that our ideas of apparent magnitude differ so much from the visual angles. This extends to all images both by reflection and refraction. The reader will find a great deal of satisfaction upon the subject in Mr. HARRIS's Optics.

58. If in the place of a real image, that is, an image formed by rays converging, a white paper be placed, the image will be formed upon the paper.

For the rays are then reflected from every point of the image in all directions, in like manner as if it were a real object. But when the paper is removed, the rays proceed only straight forward, and therefore the image can only be seen by an eye placed directly behind it.

59. The image of a circular object concentric with a single spherical refracting surface, will be also circular and concentric with it.

60. If the object be linear, its magnitude and the magnitude of the image will be as their distances from the center.

61. The image will be erect or inverted, according as it is on the same or different sides of the center in respect to the object.

62. The image of a straight line by refraction through a lens, is a conic section.

If the object be situated in the principal focus, the image is a parabola; if nearer to the lens, it is an hyperbola; if further from the lens, an ellipse.

63. If a ray of light pass out of air into any medium at an angle of incidence whose sine  $= s$ , and the sines of refraction of the least and greatest refrangible rays be  $m$  and  $n$ ; then the arc, to radius unity, measuring the whole dispersion of the rays  $= \frac{n-m}{c}$ ,  $c$  being the cosine of refraction of the mean refrangible rays.

For the same medium  $m : s :: 1 : v$  a constant ratio, and  $n : s : 1 : w$ ; hence the dispersion  $= \frac{s \times v - w}{c v w}$ . Let rays pass out of air into flint glass, and  $v = 1,565$ ,  $w = 1,595$ ; let rays also pass out of air into common glass, at the same angle of incidence, then  $v = 1,56$ ,  $w = 1,54$ ; now as we may consider  $c$  to be the same in both cases, we have the dissipating powers of these two mediums at the same angle of incidence

as  $\frac{0,03}{1,565 \times 1,595} :: \frac{0,02}{1,54 \times 1,56} :: 3 : 2$  very nearly.

When the rays fall very nearly perpendicularly, we may consider  $s$  and  $c$  as constant, and the dissipating powers will in that case, be always as  $\frac{v-w}{v w}$ .

64. If a ray of light pass through a prism and both refractions be the same way, the whole dispersion



perfection of the rays will be the sum of the dispersions at going in and coming out of the prism, each of which may be computed by the rule in the last proposition, having given the angle of incidence upon the first surface and the refracting angle of the prism.

If  $s$  = the sine of incidence on the first surface,  $c$  = cosine of refraction,  $S$  = the sine of incidence on the second surface,  $C$  = the cosine of refraction,  $v$  and  $w$  as in the last proposition; then the whole dispersion =  $\frac{s \times v - w}{c \times vw} + \frac{S \times v - w}{C \times vw} = \frac{sC + Sc}{Cc} \times \frac{v - w}{vw}$ . If the refracted ray within the prism be parallel to the base,  $s$  and  $C$  are the sines and cosines of the same angle, and so also are  $S$  and  $c$ .

65. Two prisms made of different kinds of glass may have their refracting angles so adjusted, that when the refracting angle of one is applied to the base of the other, a ray of light passing through them shall have its incident and emergent parts parallel, and the emergent part shall be coloured.

66. Two prisms may be made and applied as before, and the emergent ray shall be free from colour, but not parallel to the incident ray.

Sir I. NEWTON, after having determined the proportion of the sine of incidence to the sines of refraction of different coloured rays, as given by his glass prisms, proceeds to discover their proportions in different refracting mediums. He placed a prism of glass in a prismatic vessel of water, and refracting the light through these mediums, he found that light, as often as by contrary refractions it was so corrected that it emerged in lines parallel to those in which it was incident, continued to be white; but if the emergent rays were inclined to the incident, the light became coloured. The conclusion from this experiment was, that the divergency of the different coloured rays was constantly in a given ratio to the mean refraction in all mediums. But in the year 1757, Mr. DOLLOND tried the same experiment and found the result to be very different; for when the light was refracted in contrary directions through the glass and water prisms, if the emergent rays were parallel to the incident rays, they were found to be considerably coloured; from whence it followed that the dispersion of the

the rays of different colours was not in a constant ratio to the mean refraction in water as in glass, because there was a dispersion without any mean refraction. And further experiments proved that there was also a very considerable difference of the same kind to be found in different sorts of glass. This discovery of Mr. DOLLOND was so extraordinary, and so contrary to the best established principles, that Mr. EULER did not at first believe it. At length however Mr. ZEIHNER, at *Petersburg*, made experiments of a similar kind, and convinced Mr. EULER that it was true. He showed that it is the lead, which is used in some compositions of glass, which produces that very extraordinary property of augmenting the dispersion of the extreme rays, without sensibly changing the refraction of the mean. Mr. EULER, in a paper read at the Academy of Sciences at Berlin in 1764, was candid enough to confess that he did not at first credit the account, and thereby gave to Mr. DOLLOND the honour of the discovery. Notwithstanding this declaration of Mr. EULER, M. DE LA LANDE in his *Astronomy* published in 1764, and FUSS in his *Eulogy on EULER*, both ascribe the invention to EULER. But it still remains to be explained from whence arose this difference between Sir I. NEWTON's and Mr. DOLLOND's experiments. This Mr. P. DOLLOND has done in a pamphlet, entitled, *Some account of the discovery by the late Mr. JOHN DOLLOND, F.R.S. which led to the great improvement of refracting telescopes*. In NEWTON's time the English were not the most famous for making telescopes, and a great many were imported from *Italy*, and particularly from *Venice*. The glass made at *Venice* was nearly of the same refractive quality as our crown glass, but of better colour. It is probable that NEWTON's prisms were made of that glass, because he mentions the specific gravity of common glass to be to water as 2,58 : 1, which nearly answers to that of Venetian glass. Now Mr. DOLLOND had a piece of that glass by him, of which he made a prism, and trying the experiment with it, he found it answered very nearly to what NEWTON relates, the difference being only such as may be supposed to arise from the same kind of glass made at different times. Hence it appears, that NEWTON was accurate in his experiment, and had he used prisms of different glass, he would have made the discovery which led to the perfecting of refracting telescopes.

Mr. DOLLOND having discovered that different kinds of glass had different dispersive powers, examined all the different kinds, and found that the difference of dispersions was greatest in the *crown* and *white flint* glass. He therefore ground a wedge of flint glass at an angle of about  $25^{\circ}$ , and several others of crown glass, till he found one with the same dispersive power as the flint glass, and applying these together so as to refract in contrary directions, he found that the emergent rays were free from colour but not parallel to the incident rays. They were free from colour because the dispersive powers were equal and contrary, but not parallel to the incident rays, because the mean refractive power of each prism was different. In like manner he found that he could apply a wedge of crown glass to the flint which should have the same mean refractive but a different dispersive power, by which means the emergent rays would be parallel to the incident, but  
would

would be coloured. They would be parallel because the mean refractive powers were equal and contrary, and coloured because the dispersive powers were unequal. Thus he could produce refraction without dispersion, and dispersion without refraction.

Mr. DOLLOND next considered, that as a ray might be refracted free from colour through a wedge, it might also through a lens. When an image of an object is formed by a convex lens, it appears coloured, owing to the dispersion of the rays by refraction; as therefore rays can be refracted without dispersion by prisms, he conceived that it might also be done by a combination of lenses. And in this he succeeded, by considering, that in order to make two spherical glasses that should refract the light in contrary directions, as in the two wedges, one must be concave and the other convex; and as the rays are to converge to a real focus, the excess of refraction must be in the convex lens, because that makes rays converge and the concave makes them diverge. Also, as the convex lens is to refract most it must be made of crown glass, as appeared from the experiments with the wedges, and the concave of white flint glass. Farther, as the angle of dispersion varies inversely as the focal length, very nearly, from the principles of optics, and the angle of dispersion so varies as the dispersing powers, therefore if the focal lengths be taken inversely as the dispersing powers, found from the two wedges, the angles of dispersion will be equal, and being in contrary directions they will correct each other and the different refrangibility of light will be removed. Upon this principal Mr. DOLLOND, was enabled to make a combined lens to form an image free from colour, and therefore brought to perfection the refracting telescope, making it represent objects with great distinctness, and in their true colours. Instead of forming the object glass with one convex lens of crown and one of flint glass, two convex lenses of crown are used and the concave one of flint put between them. This construction of the object glass tends also to correct the error arising from the spherical form of the lens; for as the rays at the edge of the convex lens tend to a focus nearer to the lens than those at the middle, the concave lens, which makes the rays at the edge diverge more than those at the middle, will counteract the above effect, and bring the rays at all distances from the center of the lens to a focus more nearly together; and by a proper adjustment of the foci the diffusion of rays at the focus may be rendered inconsiderable. Telescopes thus constructed are called *Achromatic*.

#### ON THE CONSTRUCTION OF OPTICAL INSTRUMENTS.

67. A single microscope is formed by one convex lens, having the object in the principal focus, and the linear magnifying power is equal to the least distance



distance at which the naked eye can see distinctly divided by the focal length of the lens.

In this, and also in the next proportion, we compare the magnitude seen through the glass with that seen without it at the least distance of distinct vision, which is usually about 6 or 7 inches.

68. A compound microscope is formed with two convex lenses; the object is placed a little beyond the principal focus of one, by which means a large inverted image is formed, and the eye glass is placed at the distance of its focal length from the image; by this combination, the linear magnifying power is equal to the least distance of distinct vision multiplied by the distance of the image from the object glass, divided by the distance of the object from the object glass multiplied by the focal length of the eye glass.

The object appears inverted because the eye looks at an inverted image. The brightness of the object is as the magnitude of the object glass; and the field of view as the magnitude of the eye glass, all the other circumstances being the same.

69. A solar microscope is formed by two convex lenses, one of which is to receive the rays of the sun and throw them upon the object to illuminate it; and the object being situated a little beyond the principal focus of the other lens, a large inverted image is formed and received upon a screen, and magnified, in linear dimensions, as the distance of the image from the lens divided by the distance of the object from the lens.

70. A magic lanthorn is formed by placing an object a little beyond the principal focus of a convex lens, and receiving the image upon a screen; it therefore magnifies as the solar microscope.

The

The object is illuminated by a lamp or candle, and the rays are generally received on the plane side of a glass segment of a sphere and refracted to the object, in order to illuminate it more strongly.

71. A camera obscura is a box with a convex lens put into a moveable tube, by which the images of distant objects are formed upon a plane in the box proper to receive them.

This should be used when the sun shines, otherwise the image will be faint.

72. The astronomical telescope consists of a double convex lens object glass and eye glass, and the magnifying power is equal to the focal length of the object glass divided by that of the eye glass.

The distance of the two glasses = the sum of their focal lengths; and an inverted image being formed by the object glass is looked at by the eye glass. This is only used for astronomical observations, because the objects appear inverted, which, for such purposes, is of no consequence.

The brightness of the object is as the magnitude of the object glass; and the visible area as the magnitude of the eye glass, the other circumstances remaining the same. By proportioning the focal distances of these glasses, you may magnify as much as you please; but unless you can increase the quantity of light in proportion, the object will become indistinct for want of light. Now you can increase the quantity of light only by increasing the magnitude of the object glass, which you cannot do, if single, to a great degree without the object appearing coloured from the different refrangibility of the rays of light. Here then is the great use of the achromatic object glass, which admits of being very large without separating the colours, by which you can form a very perfect strong image.

73. The terrestrial telescope is composed of an object glass of a double convex lens, and three eye glasses, generally of the same focal length, and magnifies, in that case, as the astronomical telescope.

The object glass first forms an inverted image, and then the two eye glasses next to it receive the rays and invert the image again, and the third eye glass is to look at this erect image. Hence the object appears erect, and therefore is fit for terrestrial observations. In the best telescopes the object glass is achromatic, and forms a colourless image; but it might here be expected that the second image formed by the eye glasses, they not being achromatic, would be coloured, which is not the case; the reason is, that the aperture of the eye glasses is but small,

and the dispersion of the rays in the focus is as the aperture, and therefore here is but small, and that may be removed by a proper adjustment of the eye glasses, and increasing their number. See Mr. RAMSDEN's Paper on this subject in the *Phil. Transf.* 1783.

74. GALILEO's telescope is formed by a convex lens object glass, and concave eye glass, whose distance is the difference of their focal lengths, and the magnifying power is equal to the focal length of the former divided by that of the latter.

The eye glass intercepts the rays before the image is formed by the object glass, and therefore the object appears erect. The brightness is as the magnitude of the object glass; and the visible area is as the magnitude of the pupil of the eye, and will be greater also the nearer the eye is to the glass, all other circumstances being the same.

75. Sir ISAAC NEWTON's reflecting telescope is formed by placing a concave reflector to receive the rays and form the image; but those rays are received after reflection, before the image is formed, upon a plane reflector making an angle of  $45^\circ$  with the axis of the reflector, by which means the image is thrown out of the axis and lies parallel to it, where it is looked at by an eye glass. The magnifying power of this telescope is the focal length of the reflector divided by that of the eye glass.

The brightness is as the magnitude of the reflector; and the field of view as the magnitude of the eye glass, all other circumstances being the same.

This telescope was constructed by Sir I. NEWTON in order to form an image free from colour, which at that time could not be done by refraction. Since therefore the invention of the achromatic telescopes, these have been of less use; for the image, although free from colours, is not so sharp and distinct as by refraction. Instead of a plane speculum to reflect the light, a right angled prism is sometimes used, so that the rays enter and go out of the two sides including the right angle perpendicularly, and are reflected at the third side, which reflection is stronger than from a plane speculum. The third side is not quicksilvered over, for without quicksilver it will reflect all the light incident upon it from the speculum.

Since



Since the time of Sir I. NEWTON, reflecting telescopes have been differently constructed. Dr. GREGORY formed the image by a concave reflector, and then at a little distance beyond the image from that reflector he placed another concave reflector which formed a second image, which is viewed through a hole in the center of the first reflector by a convex eye glass. Mr. CASSEGRAIN afterwards made a small alteration in the construction, by placing a small convex reflector to receive the rays before the image was formed by the first reflector, by which he formed an image which was viewed as before. This construction has some advantage over the former, for as one reflector is convex and the other concave, the error arising from the spherical form of the first reflector is partly corrected by the second; whereas in the other form it would be increased.

76. When a short sighted person uses a telescope, he must push the eye glass nearer to the object glass; the contray for a long sighted person.

77. The density of light varies inversely as the square of the distance from the point from which it diverges.

Hence the quantity of light received by a telescope from an object varies inversely as the square of the distance of the object from the telescope, and consequently the brightness of the object seen through it must vary in the same ratio, every thing else being the same. By density we do not here mean the number of particles in a given space of three dimensions, but the number on a given plane.

#### ON THE RAINBOW.

78. If a ray of light enter a sphere of a denser medium, and after  $n$  reflections within it emerge; then if  $a$  = the angle of incidence,  $b$  = the angle of refraction at the entrance, the angle under the incident and emergent rays  $= 2a - 2n + 2 \times b$ .

When the number of reflections is odd, it is the angle between the rays; when the number is even, it is the angle on the other side.

79. When the angle under the incident and emergent rays is a maximum or minimum, the tangent of the angle of incidence : the tangent of refraction  $:: n + 1 : 1$ .

As the medium, in the following propositions, is supposed to be water, the sine of incidence : the sine of refraction for the least refrangible rays as 77 : 50, and for the greatest as 78 : 50.

80. The ratio of the sines and tangents of incidence and refraction being given as in the two last propositions, the angles themselves may be found.

81. The rainbow is formed by the refraction and reflection of the sun's rays on falling drops of rain.

82. Rays are said to be efficacious when a sufficient number of any one colour comes to the eye to excite the idea of that colour.

83. Rays are most efficacious when those of the same colour emerge parallel.

For then they keep together and all enter the eye. When the rays emerge parallel, they will, when produced to meet the incident rays, make equal angles with them.

84. If they make equal angles with each other, that angle must be either a maximum or minimum.

For when a quantity is either a maximum or minimum, it is at that time neither in an increasing or decreasing state.

85. Hence the angle of incidence found in prop. 80. is the proper angle to render the rays efficacious.

86. As the rays of different colours have different degrees of refrangibility, the angle of incidence, to render the rays efficacious, must be different for the different colours.

87. The primary bow is caused by two refractions and one reflection, and all the rays are reflected from the same point.

88. The

88. The secondary bow is caused by two refractions and two reflections, and all the rays from the first to the second reflection go parallel.

Hence as some rays are lost by refraction at every reflection, the primary bow is brighter than the secondary.

89. In the primary bow the least refrangible rays are uppermost; and in the secondary bow the most refrangible are uppermost.

Hence the order of the colours in the two bows is inverted.

Red is the least refrangible ray, and then orange, yellow, green, blue, indigo, violet; this therefore is the order of the colours from the upper to the under side of the primary bow, and from the under to the upper side of the secondary.

90. The bow appears circular, for the eye is in the vertex of a cone, in the surface of which all the drops lie which render the rays of any one colour efficacious.

The angle which the side of the cone makes with the axis is equal to the angle under the incident and emergent rays when efficacious; this angle is called the semidiameter of the bow, and may be computed, by first computing the angles of incidence and refraction when the rays are efficacious by prop. 80. and then prop. 78. will give the angle. In the primary bow, the angles for the least and greatest refrangible rays are  $42^{\circ}.2'$  and  $40^{\circ}.17'$ ; and for the secondary, they are  $50^{\circ}.57'$  and  $54^{\circ}.7'$ . The difference of these respective angles would be the breadth of each bow if the sun were a point; but as it is not, the breadth will be increased by the breadth of the sun, or  $32'$ . Hence the breadth of the primary bow is  $2^{\circ}.15'$  and that of the secondary  $3^{\circ}.42'$ . The axis of the cone is directed to the sun, and hence the sun is directly opposite to the center of the bow.

91. The semidiameter of the bow is equal to the altitude of its highest point added to the altitude of the sun.

Hence the altitude of the highest point is equal to the semidiameter of the bow diminished by the altitude of the sun. The primary bow therefore cannot be seen unless the sun's altitude is less than  $42^{\circ}.2'$  nor the secondary unless it is less than  $54^{\circ}.7'$ . When the sun is in the horizon each bow appears a semicircle, because the center then lies in the horizon; the bow therefore never can appear larger.



## ON THE EYE AND VISION.

92. The eye is formed of several mediums, which have the power of forming, upon optical principles, the images of objects before it upon the back part; the formation of which, in such a situation, is the cause of vision.

The eye is perfectly globular, except that the fore part is a little more convex than the rest. It consists of three mediums, or humours; that in the front is a transparent fluid like water, and is therefore called the *aqueous* humour; the next is called the *crystalline* humour, and is an hard substance like the white of an egg boiled, and in the form of a double convex lens having its back surface of the greatest curvature; the hindermost is called the *vitreous* humour, and is somewhat like to soft jelly. The whole globe, except the fore part, is surrounded by three coats; the outermost is called the *sclerotica*; the next the *choroides*, and the innermost the *retina*. The coat of the more prominent part before is called the *cornea*, being like horn, and is perfectly transparent. Adjoining to the choroides, on the front of the eye, and in the aqueous humour, is an opaque membrane called the *uvea*, in the middle of which there is a hole, called the *pupil*, for the admission of light; this the eye has the power of contracting or enlarging for the admission of less or more light, as the circumstances of vision may require. The crystalline humour is suspended by a muscle, called the *processus ciliares*, and sometimes the *ligamentum ciliare*. The several coats and surfaces of the humours are so situated as to have one straight line, called the *axis* of the eye, perpendicular to them all. At the bottom of the eye, a little towards the nose, there is a nerve which goes to the brain, called the *optic* nerve. This is a continuation of the retina. The nerves from each eye meet before they come to the brain.

Now the refractive powers of the aqueous and vitreous humours have been found by experiment to be about the same as common water, and that of the crystalline is a little greater: that is, the sine of incidence to refraction out of air into the aqueous humour is as 4 : 3, out of the aqueous into the crystalline as 13 : 12, and out of the crystalline into the vitreous as 12 : 13. Hence the aqueous and the vitreous humours being supposed to have the same refractive power, may be conceived to form one medium in which the crystalline humour is situated; the rays are therefore first refracted at the cornea into this medium and are made to converge; and then falling upon the crystalline humour, or convex lens, are made to converge more, and come to a focus at the bottom of the eye; the whole may therefore be considered as a kind of compound lens. That the pictures of all objects are formed upon the bottom of the eye appears from hence, that if the sclerotica at the bottom of the eye be taken off, the pictures of the objects before the eye will appear on the bottom.

A table

A table of the dimensions of the human eye at a medium.

|                                                                  | in,  |
|------------------------------------------------------------------|------|
| Diameter from the cornea to the choroides                        | .95  |
| Radius of the cornea                                             | .335 |
| Distance of the cornea from the first surface of the crystalline | .106 |
| Radius of the first surface of the crystalline                   | .331 |
| Radius of the back surface of the crystalline                    | .25  |
| Thickness of the crystalline                                     | .373 |

93. Having given the focus of incidence of rays upon the eye, the refraction of the mediums, and the radii of the cornea and each surface of the crystalline humour, the focus after refraction may be found.

For the focus of rays refracted at the cornea may be found by prop. 38. and that focus is the focus of rays upon the crystalline lens; hence by prop. 44. the focus after refraction by the lens may be found.

If vision arise from the formation of the image upon the retina, or, as some imagine, upon the choroides, it is manifest that a different conformation of the eye is necessary for distinct vision at different distances. Some think it is a change in the length of the eye; others a change in the figure or position of the crystalline humour; others that it is a change in the cornea. Any of these changes would produce the effect, and sufficient experiments have not yet been made to determine with certainty which is the true opinion. As the rays suffer a greater refraction at the cornea, than they do afterwards, it is manifest that a less change in the radius of the cornea will effect the business, than will suffice in any other part of the eye.

94. A long sighted person must use a convex lens to see a near object distinctly; and a short sighted person must use a concave lens to see a distant object distinctly.

A long sighted person has either the cornea or crystalline humour, or both, too *flat*, and therefore the image of a near object would be formed beyond the bottom of the eye; this may be corrected by a convex lens, which makes rays converge more, and thereby form the image on the bottom of the eye. But a short sighted person has them too *spherical*, and therefore the image is formed before the rays come to the bottom of the eye; to correct this, and form the image on the bottom, a concave lens must be used, which makes rays diverge. If  $m$  be the distance at which either person can see distinctly with the naked eye, and  $n$  the distance at which he wants to see with the glass, then

$\frac{mn}{m+n}$  is the focal length of the glass. Those who use glasses should have

have them as accurately as possible adapted to the eye, otherwise they will strain the eye. It is a common opinion that looking through glasses is detrimental to the eye; but this is so far from being true, that, to preserve the eye, those who want them ought to use them, otherwise the eye will continually be subject to be strained by endeavouring to see objects distinctly. Also opticians, who are continually examining telescopes, find no inconvenience from it.

95. The vibrations communicated by the optic nerve to the brain, from the impulses of the rays of light upon the bottom of the eye, are supposed to cause the sense of seeing.

This is the opinion of Sir I. NEWTON. He supposes also that the several sorts of rays make different vibrations, which accordingly excite sensations of different colours, much after the same manner that different vibrations of air excite sensations of different sounds. The flashes of lightning sometimes perceived by a blow upon the eye, or the colours from pressing the eye sideways, are probably owing to the same kind of vibrations being excited, as by the impressions of light. Such phenomena as these tend to confirm the hypothesis, that vision is caused by some motion excited in the optic nerve, by the impulse of light. The ideas we have from the impressions of light remain for a small time, as is manifest from the phenomenon of a burning coal appearing like a ring of fire, when whirled swiftly round. The stronger the light is the longer the sensation remains, as appears from looking at the sun, in which case the sensation will continue some minutes. All these circumstances render it very probable that the sensation arises from a vibrating motion.

Sir I. NEWTON supposes that every point of the retina of one eye, hath its corresponding point on the other; from which two slender pipes filled with a liquid go along the optic nerve, and meeting before they come to the brain, their joint effect produce but one sensation. Hence if an eye be distorted, objects appear double, because corresponding points of the image do not fall upon these corresponding points of the retina. When we look directly at an object, the axis of both our eyes are directed to it, and the corresponding points of each image agree with those of the retina, and the object therefore appears single; but at the same time any other object in the same line either nearer or further off appears double, because corresponding points of each image do not, in this case, fall on those of the retina. If you shut one eye, the object not looked at directly then appears single. In a matter however of so much uncertainty, it is no wonder that different authors have invented different hypotheses.

When the image of an object falls upon the optic nerve, the object becomes invisible to that eye. Hence an object cannot become invisible to both eyes at the same time, because the image cannot fall upon the optic nerve of each eye at the same time. An object seen with both eyes appears about  $\frac{1}{10}$  or  $\frac{1}{12}$  brighter than with one eye.

The



The angle subtended by the least visible object cannot be very accurately ascertained, as it depends upon the colour of the object, and the ground upon which it may be seen; it depends also upon the eye. Mr. HARRIS thinks the least angle for any object to be about  $40''$ ; and at a medium about  $2'$ .

To the generality of eyes the nearest distance of distinct vision is about 7 or 8 inches. Hence if we assume 7 inches for that distance, and 2 minutes for the least visible angle, a globular object of less than about  $\frac{1}{250}$ th part of an inch cannot be seen. See HARRIS's Optics upon these subjects.

96. The eye, as to sense, corrects the different refrangibility of the rays of light.

For objects seen by the naked eye are not tinged with the prismatic colours. EULER supposed the eye to be perfectly achromatic. Dr. MASKELYNE in the *Phil. Trans.* 1789, has examined this point, and taking the dimensions of the eye from Mr. PETIT, and the refractive powers of the different mediums from Mr. HAUKEBEE, has computed the diameter of the circle of aberration upon the retina, and found it to be .002667 of an inch; a quantity too small to be perceived. He thinks some such an angle of aberration as this is necessary in order to account for the sensible diameters of some of the fixed stars.

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# A S T R O N O M Y.

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## D E F I N I T I O N S.

1. A *GREAT* circle of a sphere is that whose plane passes through its center; and a *small* circle is that whose plane does not pass through its center.

2. A diameter of a sphere perpendicular to any great circle, is called the *axis* of that circle, and the extremities of the diameter, are called its *poles*.

Hence the pole of a great circle is  $90^\circ$  from every point of it upon the surface of the sphere; but as the axis is perpendicular to the circle when it is perpendicular to any two radii, therefore a point on the surface of a sphere  $90^\circ$ . distant from any two points of a great circle will be the pole.

3. All angular distances on the surface of a sphere to an eye at the center, are measured by the arcs of great circles; for then being arcs to equal radii, they will be as the angles.

Hence all triangles formed upon the surface of a sphere for the solution of spherical problems, must be formed by the arcs of great circles.

4. All great circles must bisect each other; for passing through the center of the sphere their common section must be a diameter, which bisects all circles.

5. Secondaries to a great circle are great circles which pass through its poles.

Cor. 1. Hence secondaries must be perpendicular to their great circle; for if one line be perpendicular to a plane, any plane passing through that line will also be perpendicular to it; hence as the axis of the great circle is perpendicular to it, and is the common diameter to all the secondaries, they must all be perpendicular to the great circle. Hence also every secondary must bisect its great circle, and every small circle parallel to it; for the plane of the secondary passes through not only the center of the great circle, but also of the small circles parallel to it.

Cor. 2. Hence a great circle passing through the poles of two great circles, must be perpendicular to each; and, vice versâ, a great circle perpendicular to two other great circles must pass through their poles.

6. A circle appears a straight line to an eye in its plane; hence in the representation of the surface of a sphere upon a plane, those circles whose planes pass through the eye are represented by straight lines.

7. The

7. The angle formed by two great circles on the surface of a sphere is equal to the angle formed by the planes of the circles; and is measured by the arc of a great circle intercepted between them described about the intersection of the circles as a pole.

For let  $C$  (fig. 7.) be the center of the sphere,  $PQE$ ,  $PRE$  two great circles; draw the tangents  $Px$ ,  $Pz$ , then the angle  $xPz$  = the angle formed by the two circles; and as these tangents are perpendicular to the common intersection  $PCE$ , the angle between them is equal to the angle between the planes, by Eu. B. 11. def. 6. Now draw  $CQ$ ,  $CR$  perpendicular to  $PCE$ ; then the angle  $QCR$  is the angle between the planes, and therefore equal to the angle formed by the two circles, and this angle is measured by the arc  $QR$  of a great circle, which arc has for its pole the point  $P$  by def. 2. because  $PQ$ ,  $PR$  are each  $90^\circ$ .

8. If at the intersection of two great circles as a pole, a great circle be described, and also a small circle parallel to it; the arcs of the great and small circles intercepted between the two great circles contain the same number of degrees.

For draw  $AB$ ,  $AD$  perpendicular to  $PCE$ , then as  $AB$ ,  $AD$  are parallel to  $CQ$ ,  $CR$ , the plane  $ABD$  is parallel to the plane  $QCR$ , and therefore the small circle  $BD$  of which  $A$  is the center is parallel to the great circle  $QR$ ; and as each angle  $BAD$ ,  $QCR$ , measures the inclination of the planes they must be equal, and consequently the arcs  $BD$ ,  $QR$  contain the same number of degrees. Hence the arc of such a small circle measures the angle at the pole between the two great circles. Also  $QR : BD :: QC : BA :: \text{radius} : \cos. BQ$ .

Cor. Hence  $QR$  is the greatest distance between the two circles; and if from  $R$ , a point  $90^\circ$ . from  $P$ , a great circle  $QR$  be drawn perpendicular to  $PQ$ , the arc  $RQ$  is the measure of the angle at  $P$ .

9. The *axis* of the earth is that diameter about which it performs its diurnal motion; and the extremities of this diameter, are called its *poles*.

10. The *terrestrial equator* is a great circle of the earth perpendicular to its axis. Hence the axis and poles of the earth are the axis and poles of its equator. That half of the earth which lies on the side of the equator which we inhabit, is called *north*, and the other *south*; and the poles, are called the north and south poles.

11. The *latitude* of a place on the earth's surface, is an arc measured from the equator upon a secondary to it.

12. The *longitude* of a place on the earth's surface, is the arc upon the equator between a secondary to it passing through the place, and another secondary passing through any other place from which you begin to measure.

13. If the plane of the terrestrial equator be produced to the sphere of the fixed stars, it marks out a circle, called the *celestial equator*; and if the axis of the earth be produced in like manner, the points in the heavens to which it is produced, are called poles, being the poles of the celestial equator. The star nearest to each pole, is called the *pole star*.

14. Secondaries to the celestial equator, are called circles of declination; because the declination of an heavenly body is its angular dis-



tance from the equator measured upon a secondary to it. Of these, 24, which divide the equator into equal parts, each containing  $15^\circ$ , are called *hour circles*.

15. Small circles parallel to the equator, are called *parallels of declination*.

16. The *sensible horizon*, is that circle in the heavens which bounds the spectator's view. The *rational horizon*, is a great circle in the heavens passing through the earth's center parallel to the sensible horizon.

17. If the radius of the earth to the place where the spectator stands be produced both ways to the heavens, that point vertical to him, is called the *zenith*, and the opposite point the *nadir*.

Hence the zenith and nadir are the poles of the rational horizon.

18. Secondaries to the horizon, are called *vertical circles*, because they are perpendicular to the horizon, by def. 5. cor. 1.; on these circles therefore the altitude of an heavenly body may be measured.

19. A secondary common to the equator and horizon, and which therefore passes through the poles of each, by def. 5. cor. 2. is called the *meridian*.

20. The meridian of any place divides the heavens into two hemispheres, one of which is called the *eastern*, and the other the *western* hemisphere.

21. To a spectator on the north side of the equator, that direction which passes through the north pole, is called *north*, and the opposite direction *south*; hence the meridian which passes through the zenith of the spectator and through the poles, must cut the horizon in the *north* and *south* points.

22. A vertical circle which cuts the meridian of any place at right angles, is called the *prime vertical*; and the points where it cuts the horizon, are called the *east* and *west* points.

Hence the east and west points are  $90^\circ$  distant from the north and south. These four, are called the *cardinal points*.

23. The *azimuth* of an heavenly body is its distance on the horizon, when referred to it by a secondary, from the north or south points. The *amplitude* is its distance from the east or west.

24. Small circles parallel to the horizon are called *almicantars*.

25. The *ecliptic* is that great circle in the heavens, which the sun appears to describe in the course of a year.

26. The ecliptic and equator being great circles, the points where they bisect each other are called the *equinoctial points*.

27. The ecliptic is divided into 12 equal parts, called *signs*; Aries  $\gamma$ , Taurus  $\tau$ , Gemini  $\Pi$ , Cancer  $\♋$ , Leo  $\Omega$ , Virgo  $\♍$ , Libra  $\♎$ , Scorpio  $\♏$ , Sagittarius  $\♐$ , Capricornus  $\♑$ , Aquarius,  $\♒$ , Pisces  $\♓$ . The order of these is according to the motion of the sun. The first point of Aries coincides with one of the equinoctial points, and the first point of Libra with the other.

28. The motion of the heavenly bodies according to the order of the signs, is called *direct*, or *in consequentia*; and the motion in the contrary direction, is called *retrograde*, or *in antecedentia*.

The *real* motion of all the planets is according to the order of the signs, but their *apparent* motion is sometimes in a contrary direction.

29. The

29. The *right ascension* of a body is an arc of the equator between the first point of Aries, and a declination circle passing through the body, measured according to the order of the signs.

30. The *oblique ascension*, is the distance from the first point of Aries, to that point of the equator which rises with any body, measured according to the order of the signs.

31. The *ascensional difference*, is the difference between the *right* and *oblique* ascension.

32. The *longitude* of an heavenly body, is an arc of the ecliptic between the first point of Aries, and a secondary to the ecliptic passing through the body, measured according to the order of the signs.

33. The *latitude* of an heavenly body, is its angular distance from the ecliptic, measured upon a secondary to it drawn through the body. If that angle be seen from the earth, it is called the *geocentric* latitude, but as seen from the sun, it is called the *heliocentric* latitude.

34. The *equinoctial colure*, is a secondary to the equator passing through the equinoctial points: the *solstitial colure*, is a secondary common to the ecliptic and equator. Hence the solstitial colure passes through the poles of the equator and ecliptic, by def. 5.

35. The *tropics* are two parallels of declination touching the ecliptic. One touching it at the beginning of Cancer, is called the tropic of Cancer; and the other touching it at the beginning of Capricorn, is called the tropic of Capricorn.

36. The *arctic* and *antarctic* circles are two parallels of declination, the distance of which from the two poles, is equal to the distance of the tropics from the equator.

37. A body is in *conjunction* with the sun when it has the *same* longitude; and in *opposition*, when the difference of their longitudes is  $180^\circ$ . The conjunction is marked thus  $\odot$ , and the opposition thus  $\oslash$ .

38. The *elongation* of a body is its angular distance from the sun seen from the earth.

39. The *diurnal parallax* is the difference between the apparent places of the bodies in our system when referred to the fixed stars, seen from the center and surface of the earth.

40. The *argument*, is a term used to denote any quantity, by which another required quantity may be found. For example, the argument of that part of the equation of time, which arises from the unequal angular motion of the earth in her orbit about the sun, is the sun's anomaly, because the equation depends entirely upon the anomaly, and the latter being given, the former is immediately found. The argument of a planet's latitude is its distance from the node, because upon this the latitude depends.

41. The *nodes*, are the points where the orbits of the primary planets cut the ecliptic, and where the orbits of the secondaries cut the orbits of their primaries. That node is called *ascending* where the planet pass from the south to the north side of the ecliptic; and the other is called the *descending* node. The ascending node is marked thus  $\Omega$ , and the descending thus  $\oslash$ .

42. The angle of *commutation*, is the angle at a planet formed by two lines one drawn to the earth and the other to the sun; hence this angle

angle is the difference of the places of the planet seen from the earth and sun.

43. The angle of *position*, is the angle at an heavenly body formed by two great circles, one passing through the pole of the equator, and the other the pole of the ecliptic.

Characters used for the Sun, Moon, Planets: the Sun ☉; the Moon ☾; Mercury ☿; Venus ♀; the Earth ☷; Mars ♂; Jupiter ♃; Saturn ♄; the Georgium Sidus HL.

#### ON THE APPARENT MOTIONS OF THE HEAVENLY BODIES, THE DOCTRINE OF THE SPHERE, AND PRINCIPLES OF DIALLING.

1. The sun appears to describe the ecliptic in  $365d. 6h. 9'. 10'', 37$ ; in which time it is found to be only twice in the equator. This is called a *fidereal* year.

Cor. Hence the ecliptic is inclined to the equator.

The time the sun describes the ecliptic is found by taking the difference between its longitude and that of any fixed star, and then observing when that difference becomes the same again; and the interval of those times gives the time in which the sun appears to make a complete revolution in the heavens. If  $360^\circ$  be divided by  $365d. 6h. 9'. 10'', 37$  it gives  $59'. 8''$ , the space the sun *would* describe in one day if all the days were of the same length, or the space described in a *mean* solar day. Let a clock be adjusted to go 24 hours in a mean solar day; then as the mean increase of the sun's right ascension in 24 hours is  $59'. 8''$ . the earth in a mean solar day must describe about its axis  $360^\circ. 59'. 8''$ .; but when the earth has described  $360^\circ$ . the same fixed star returns to the meridian; hence  $360^\circ. 59'. 8'' : 360^\circ :: 24 h. : 23 h. 56'. 4''$ . the length of a *fidereal* day in *mean* solar time.

The interval of time from the sun's leaving the first point of Aries, till his return to it, is  $365d. 5h. 48'. 48''$ . This is called a *tropical* year.

The return of the sun to the first point of Aries, before it has completed its revolution amongst the fixed stars, shows that the intersection of the equator with the ecliptic has a retrograde motion, called the *precession of the equinoxes*. By comparing the place of the first point of Aries as observed by the antient astronomers with its present situation, it appears that its motion in 100 years, is  $1^\circ. 23'. 40''$ .

2. All the heavenly bodies appear daily to describe circles, coincident with, or parallel to, the equator.

This is a consequence of the earth's rotation about its axis.

The



The motions of all the bodies in our system are referred to the fixed stars, whose relative situations not being altered by our own motion, they are conceived as placed in the concave surface of a sphere having the eye in the center.

3. Those bodies which are on the same side of the equator with the spectator, continue longer above the horizon than below; those on the contrary side continue longest below.

Hence when the sun is on the same side of the equator with the spectator, the days are longer than the nights; when on the contrary side, the nights are longest. Hence the variety of seasons arises from the inclination of the ecliptic to the equator.

As the orbits of the moon and planets are also inclined to the equator, a variation of the times of their continuance above and below the horizon will also take place.

4. If a spectator be in the equator, all the heavenly bodies continue as long above the horizon as below.

Hence to a spectator at the equator the days are always 12 h. long.

5. If a spectator be at the pole, all the fixed stars appear to describe circles parallel to the horizon, the equator now coinciding with the horizon; therefore they never rise and set.

6. To a spectator at the pole, the sun appears above the horizon all the time he is on the same side of the equator with the spectator; and below the horizon all the time he is on the contrary side.

*and until he is 33 minutes below the equator.*  
Hence to a spectator at the pole, there is half a year day, and half a year night.

7. In north latitude, those bodies which have north declination rise between the east and north; and those which have south declination rise between the east and south.

8. The

8. The altitude of the pole above the horizon is equal to the latitude of the place.

If in lat.  $45^{\circ}$ . a spectator travel 69,2 miles upon the meridian towards the north, the pole will be  $1^{\circ}$ . higher. Hence, supposing the earth to be a sphere, its circumference would be 24912 miles, and radius 3964: but the figure of the earth is a spheroid, whose polar diameter : the equatorial :: 229 : 230, according to Sir I. NEWTON.

9. The latitude of a place may be found by observing the greatest and least altitude of a circumpolar star, corrected for refraction, and taking half their sum. *unless there be any diff. in the Aberr. or Neut.*

10. All those stars which are not further from the pole than the <sup>*+ the Hor. Refr.*</sup> latitude of the place, never set.

These are called circumpolar stars.

11. If the declination of a fixed star be known, the sum or difference of its <sup>*true*</sup> meridian altitude and its declination, according as the star is on the contrary or same side of the equator with the spectator, will give the latitude of the place.

12. The altitude of that point of the equator which is upon the meridian, or the inclination of the equator to the horizon, is equal to the complement of the latitude.

13. The greatest declination of the sun is equal to the inclination of the ecliptic to the equator.

14. The inclination of the equator to the ecliptic, is equal to half the difference between the sun's meridian altitudes on the longest and shortest days.

The inclination at this time is about  $23^{\circ}.28'$ ; but it keeps gradually diminishing at the rate of about  $\frac{1}{2}''$  in a year. This arises from the variation of the ecliptic, which is owing to the attraction of the planets upon

*The Polar Distance of circumpolar stars may be accurately determined by observing their greatest azimuth on each side of the pole for the Refr. does not affect them. A transit circle may also be brought into the meridian nearly by this means.*

upon the earth, by which the plane of its orbit is continually varying. The whole variation however can never exceed much more than a degree. It will decrease for a considerable time, and then increase again.

15. The angle  $a^\circ$  contained between the meridian of any place, and the <sup>hour circle or</sup> circle of declination passing through the sun, turned into time at the rate of  $15^\circ$  for an hour, <sup>or a portion of apparent time</sup> gives the time from apparent noon. *whether before or after.*

This is not accurate, because the solar days are not all equal to 24 hours, but to  $24h. \pm e$  the variation of the equation of time for that day, according as the equation is increasing or decreasing: hence to get the time more accurately, say,  $360^\circ : a^\circ :: 24h. \pm e : \text{the time}$ . The time is here supposed to be mean solar time, measured by a clock which goes 24 hours in a mean solar day. This quantity  $e$  is sometimes  $30'$ , and therefore if  $a^\circ = 60^\circ$  the correction is  $5''$ . If extreme accuracy were also required, the change of declination must also be considered, which in the moon may be considerable.

If  $Z$  (fig. 8.) be the zenith of any place,  $P$  the pole,  $S$  the sun, and these points be joined by the arcs of three great circles, then  $ZS$ ,  $ZP$ ,  $PS$ , are the complements of the sun's altitude, of the latitude and of the sun's declination respectively; also the angle  $ZPS$  is the measure of the time from apparent noon, and  $SZP$  is the azimuth from the north.

16. Given the latitude of the place, the sun's altitude and declination, to find the hour and azimuth.

By spherical trig.  $\sin. SP \times \sin. ZP : \text{rad.}^2 :: \sin. \frac{1}{2} \times \overline{SZ + SP - PZ} \times \sin. \frac{1}{2} \times \overline{SZ + PZ - SP} : \sin. \frac{1}{2} ZPS^2$ , hence  $ZPS$  is known, which converted into time at the rate of  $15^\circ$  for an hour, gives the time from apparent noon. Also  $\sin. SZ \times \sin. ZP : \text{rad.}^2 :: \sin. \frac{1}{2} \times \overline{SP + SZ - ZP} \times \sin. \frac{1}{2} \times \overline{SP + ZP - SZ} : \sin. \frac{1}{2} SZP^2$ , hence the azimuth  $SZP$  from the north is known. *These may be found with considerable accuracy by a Stereographic projection. vid. Calcul. Astron. pa. 182*

17. Given the latitude of a place, the altitude, right ascension and declination of a fixed star, to find the time.

See my *Practical Astronomy*, pag. 53, where the reader will find the rule with an example; also an example to find the time by the sun. These are the methods usually practised at sea for finding the time.



18. Given the latitude and sun's declination, to find its altitude on the prime vertical, and the time.

In this case the angle  $Z$  is a right one; hence  $\text{cof. } ZP : \text{rad.} :: \text{cof. } SP : \text{cof. } ZS$ , or,  $\text{fin. lat.} : \text{rad.} :: \text{fin. dec.} : \text{fin. of the altitude.}$  Also  $\text{rad.} : \text{cot. } PS :: \text{tan. } PZ : \text{cof. } ZPS$ , or  $\text{rad.} : \text{tan. dec.} :: \text{cot. lat.} : \text{cof. } ZPS$ , which converted into time, gives the time from apparent noon.

19. Given the latitude and sun's declination, to find the altitude at 6 o'clock.

Here the angle  $ZPS$  is a right one; hence  $\text{rad.} : \text{cof. } ZP :: \text{cof. } PS : \text{cof. } ZS$ , or  $\text{rad.} : \text{fin. lat.} :: \text{fin. dec.} : \text{fin. of the altitude.}$

20. Given the latitude and sun's declination, to find the time of its rising, and azimuth at that time.

When one side of a spherical triangle  $= 90^\circ$ , the triangle may be solved by the circular parts, just as when one angle is a right one, by taking the angles adjacent to the side of  $90^\circ$  and the complements of the other three parts for the circular parts; for if we conceive the supplemental triangle to be taken, it will have a right angle, and the sine, cosine and tangent of any arc is the same as of the supplement, regard not being had to the signs of the two latter, which is here of no consequence. This circumstance is not, that I know of, taken notice of by any writers on spherical trigonometry. Hence, as  $ZS = 90^\circ$  in this case,  $\text{rad.} : \text{cot. } SP :: \text{cot. } ZP : \text{cof. } ZPS$ , or  $\text{rad.} : \text{tan. dec.} :: \text{tan. lat.} : \text{cof. of the hour angle from apparent noon.}$  This supposes that the body is upon the rational horizon at the instant it appears; but upon account of refraction, bodies in the horizon appear  $33'$  higher than their true places; hence they become visible when they are  $33'$  below the horizon. Also bodies in our system are depressed by parallax; hence when such bodies first appear,  $ZS = 90^\circ + 33' - \text{hor. par.}$  Also  $\text{fin. } ZP : \text{rad.} :: \text{cof. } SP : \text{fin. } PZS$ , or  $\text{cof. lat.} : \text{rad.} :: \text{fin. dec.} : \text{fin. of azimuth from the north.}$

21. Given the latitude and sun's declination, to find the time when twilight begins.

Twilight begins when the sun is about  $18^\circ$  below the horizon; hence  $ZS = 108^\circ$ ; therefore  $\text{fin. } SP \times \text{fin. } ZP : \text{rad.}^2 :: \text{fin. } \frac{1}{2} \times 108^\circ + SP - PZ \times \text{fin. } \frac{1}{2} \times 108^\circ + PZ - SP : \text{fin. } \frac{1}{2} ZPS^2$ , hence  $ZPS$  is known, which converted into time, gives the time from apparent noon. Twilight is caused by the refraction of the sun's rays by the atmosphere. It is observed that the distance of the sun below the horizon when twilight ends in the evening, is greater than its distance from the horizon in the morning when it begins; it is longer also in summer than in winter.

22. As

22. As the apparent diurnal motion of the sun about the axis of the earth is at the rate of  $15^\circ$  in an hour, if the earth were transparent and the axis opaque, its shadow would revolve at the same rate, being always projected into the meridian opposite to the sun.

23. If we conceive a plane passing through the center of the earth, coinciding with the rational horizon of any place, and right lines be drawn from the center to the points where the hour circles cut that plane, they will represent the hour lines on an horizontal dial for that place.

In every dial the gnomon, when fixed, is parallel to the earth's axis, and on account of the sun's great distance compared with the radius of the earth, the apparent motion of the sun may be conceived to be the same about the gnomon as about the earth's axis, and therefore an horizontal dial may be constructed similar to the construction in the proposition. Now when the sun is in the meridian, the 12 o'clock hour circle is perpendicular to the plane, and the arc from the pole to the plane is equal to the latitude of the place, and the 1 o'clock hour circle makes an angle at the pole with it of  $15^\circ$  and forms the hypotenuse of a right angled triangle to the above perpendicular, and the base is the arc measuring the angle between the 12 and 1 o'clock line; to find which we have, by spher. trig.  $\text{rad.} : \sin. \text{lat.} :: \tan. 15^\circ : \tan. \text{of the hour angle between 12 and 1 o'clock.}$  If instead of  $15^\circ$  we put  $30^\circ$ ,  $45^\circ$ , &c. we shall get the angles between the 12 and 2, 3, &c. o'clock lines.

24. If we conceive a plane passing through the center of the earth perpendicular both to the horizon and meridian, and on the south side lines be drawn from the center to the points where the hour circles cut that plane, they will represent the hour lines on a vertical south dial.

Hence a vertical south dial may be constructed in a manner similar to this construction. In this case, the arc of the meridian from the pole to the plane, is equal to the complement of latitude; hence, for the same reason as before,  $\text{rad.} : \cos. \text{lat.} :: \tan. 15^\circ : \tan. \text{of the hour angle between 12 and 1 o'clock.}$  In like manner, as before we get

the other hour angles. Upon the same principles, the hour angles may be calculated for any other plane.

The general principles of dialling may also be explained in the following manner. Insert an axis in a cylinder; divide the circumference of one end into 24 equal parts, and draw lines from them to the center, and these lines will be the hour lines for a polar dial. From these points of division, draw lines upon the surface of the cylinder parallel to the axis, and cutting the cylinder through by any section, draw lines from these parallel lines to the center of the section, and placing the axis of the cylinder parallel to the earth's axis, you have a dial for that plane.

#### ON PARALLAX AND REFRACTION.

25. Every body appears elevated, by the refraction of the atmosphere, above its true place.

This follows from the common principles of Optics. Tab. 1. gives the refraction at all altitudes.

26. Every body appears depressed by parallax below its true place in a vertical circle; and the sine of the parallax varies as the sine of the zenith distance directly, and the distance of the body from the center of the earth inversely.

The horizontal parallax of the sun, as determined from the transit of venus, is  $8''.7$ ; hence if we take the radius of the earth = 3964 miles, we have  $\sin. 8''.7 : \text{rad.} :: 3964 : 94000474$  miles, the sun's distance. The real distance of the sun being thus known, and their relative distances from their periodic times, the real distances of all the planets will be known.

The distance of the fixed stars is so great that they have no diurnal parallax. It appears also that they have no annual parallax.

When the altitude of a body is observed, it must be corrected by parallax and refraction, adding the former, and subtracting the latter, in order to get the true altitude, or the altitude above the rational horizon at the center of the earth.

#### ON PRACTICAL ASTRONOMY, AND THE INSTRUMENTS FOR THAT PURPOSE.

27. The **VERNIER** is a graduated index moveable against the arc of a quadrant, or any graduated



ated line, in order to subdivide it to a greater degree of accuracy than could be done by an actual subdivision.

The principal is this: if equal arcs  $A$  of the quadrant and its index be divided, one into  $n$  and the other into  $n+1$  equal parts, the difference of the lengths of each division  $= \frac{n+1A - nA}{n \times n+1} = \frac{A}{n \times n+1}$ . If  $A=70^\circ$ , and each

degree be divided into 3 equal parts, then  $A$  is divided into 21 equal parts; and if an arc  $A$  of the index be divided into 20 equal parts, the

difference of each division  $= \frac{70}{20 \times 21} = 1'$ . Hence when any two di-

visions coincide, the distance of the next two  $= 1'$ , of the next two  $= 2'$ , of the next two  $= 3'$ , &c. This method of subdivision is applied to most quadrants.

28. The TRANSIT TELESCOPE is a telescope moveable about an horizontal axis, and so adjusted as to make its line of collimation describe a great circle passing through the pole and zenith, or the meridian of the plane.

The line of collimation is the line joining the center of the object glass and the center of the cross wires in its principal focus. One of the cross wires passing through the center is perpendicular to the horizon, and consequently it coincides with the meridian. Hence when any body comes to this wire it is in the meridian. The use of this instrument is to take the right ascensions of the heavenly bodies, and to correct the going of the clock.

A *sidereal* day is the interval between the two successive passages of a fixed star over the meridian.\* A *solar* day is the interval between the two successive passages of the sun's center over the meridian; these days are not all equal. If we conceive the year to be divided into the same number of days of equal lengths, such a day, is called a *mean solar* day. A clock adjusted to go 24 hours in a *sidereal* day, is said to be adjusted to *sidereal* time. If it be adjusted to go 24 hours in a *mean solar* day, it is said to be adjusted to *mean solar* time.

29. The interval of time between the two successive passages of a fixed star over the meridian : the interval between the passages of two fixed stars ::  $360^\circ$  : the difference of their <sup>apparent</sup> right ascensions.

Hence if we know the right ascension of one star, we can find the right ascension of all the others. The method of first finding the right ascension

ascension of a star is explained in my *Practical Astronomy*, p. 86. If we thus compare the right ascension of a known fixed star with that of the sun, moon, or a planet, we shall get their right ascensions. A clock adjusted to sidereal time, is that by which we determine right ascensions; transit clocks in observatories are therefore thus adjusted.

30. If  $x$  be the difference of the sun's and a planet's motion in right ascension in 24 hours, reduced into time,  $t$  the difference of their right ascensions in time when the sun is on the meridian,

then  $\frac{24t}{24 \pm x}$  is the time from apparent noon when

the planet is on the meridian, where the upper or lower sign prevails, according as the planet's or sun's motion is the greatest. If the planet be retrograde,  $x$  must be the sum of the motions of the sun and planet with the sign  $+$ .

As  $\frac{24t}{24 \pm x} = t \pm \frac{tx}{24} + \frac{tx^2}{24^2} \pm \&c.$  the two first terms will be sufficiently exact for all cases; except that of the moon, where it will be necessary to take the next term. The values of  $t$  and  $x$  may be put down in decimals, by tab. 2. Apparent noon is the time when the sun's center is on the meridian.

Ex. On July 1, 1767, the sun's AR when on the meridian of Greenwich, was  $6h.40'.25''$ , and its daily increase  $4'.8''$ ; also the moon's AR, was  $10h.36'.8''$ , and its daily increase  $42'.28''$ , to find the time of the moon's passage over the meridian. Here  $t = 10h.36'.8'' - 6h.40'.25'' = 3h.55'.43'' = 3,9285$ , also  $x = 42'.28'' - 4'.8'' = 38'.20'' = .6388$ ; hence  $\frac{tx}{24} = 6'.16''$ ,  $\frac{tx^2}{24^2} = 10''$ ; therefore  $3h.55'.43'' + 6'.16'' + 10'' = 4h.2'.9''$ . the time from apparent noon. If the equation of time be applied, it gives the time by the clock.

31. The ASTRONOMICAL QUADRANT is an instrument for measuring the altitudes of the heavenly bodies above the horizon.

Some quadrants turn upon a vertical axis, by which the altitudes of bodies in any situation may be measured; others, called *mural* quadrants, are fixed against a wall with their plane in the meridian, with which you can only measure meridian altitudes. Instead of a quadrant for

for taking altitudes, Mr. RAMSDEN has invented a new instrument, called a *circular instrument*, which has many advantages over that of the quadrant. A description of this may be seen in my *Practical Astronomy*. After the altitude is taken it must be corrected for parallax and refraction, by which you get the true altitude above the rational horizon.

32. The latitude of the place and the meridian altitude of a body being known, its declination will be known.

33. The right ascension and declination being known, the place of the body in the heavens is known.

34. Given the right ascension and declination of an heavenly body, its latitude and longitude may be computed.

The best rule for this purpose is that given by Dr. MASKELYNE, which the reader may see in my *Practical Astronomy*, pag. 113.

The foundation of all astronomy is to determine the situation of the fixed stars, in order to refer the places of other bodies to them from time to time, and from thence to determine their proper motions.

35. If equal altitudes of an heavenly body be taken on different sides of the meridian, the middle point of time between will give the time when the body is upon the meridian, if it have not changed its declination.

The correction for the variation of declination may be seen in my *Practical Astronomy*, pag. 44. By this means the time when any body comes to the meridian may be found; and when applied to the sun or a fixed star, the rate at which a clock, adjusted to mean solar or sidereal time, gains or loses may be determined.

36. If a small error  $m$  be made in taking the altitude of the sun or a star; the corresponding error of time will be equal to  $m \times \frac{\text{rad.}^2}{\cos.\text{lat.} \times \sin.\text{azim.}}$ ; the time being found by prop. 16.

The



The investigation of this may be seen in my *Practical Astronomy*, pag. 42.

Hence when the latitude is given the error in time is the least upon the prime vertical, and is independent of the declination. To find the time therefore from an observed altitude of the sun or a star, it should be taken upon the prime vertical, as being subject to the least error. In lat.  $52^{\circ}.12'$ , if the error in altitude at an azimuth  $44^{\circ}.22'$

be  $1'$ , the error in time  $= 1' \times \frac{1^2}{.613 \times .699} = 2'.334$  of a degree  $= 9''.336$  the error of time.

37. The EQUATORIAL INSTRUMENT consists of three circles, the azimuth circle, the equatorial circle and the declination circle; the first may be adjusted parallel to the horizon, the second parallel to the equator and the third perpendicular to the equator.

The declination circle has a telescope applied to it, whose line of collimation is parallel to that circle.

The uses of this instrument are — to take the altitude of a body above the horizon — to determine the position of the meridian — to determine the time of the day — to find a star or planet in the day time — to find the right ascension and declination of a star, and to measure horizontal angles.

38. The EQUATORIAL SECTOR is an instrument for taking the difference of the right ascensions and declinations of stars.

There are various constructions of this instrument; Mr. GRAHAM constructed the first; afterwards Dr. MASRELYNE constructed one upon different principles, which had many advantages over that by Mr. GRAHAM.

39. The ZENITH SECTOR is an instrument constructed for the purpose of measuring small angular distances from the zenith.

This instrument was invented by Dr. HOOK, in order to determine the annual parallax of the fixed stars, as, upon account of the length of its radius, it is capable of measuring small angles with greater accuracy than the quadrant. It was with this instrument, that Dr. BRADLEY made his two admirable discoveries of the *aberration of light* in the fixed stars, and the *nutation* of the earth's axis.

40. A MICROMETER is an instrument invented at first to measure the angular distances of such bodies as appear in the field of view of a telescope at the same time, or the diameters of the sun, moon or planets. But the use was afterwards extended to measure the distances of bodies more remote from each other.

41. HADLEY'S QUADRANT is an instrument for measuring the altitudes of bodies above the horizon, or the angular distance of any two bodies, whatever be their position.

As this instrument is constructed to measure the angular distance of any two bodies, and as it will do this even when the observer is subject to any unsteadiness, it is extremely well adapted to find the longitude at sea, by the moon's distance from the sun or a fixed star. With a good instrument the time found from taking the altitude of the sun or a star at land, may be depended upon to about 2" in our latitude, if the altitude be taken on or near the prime vertical. Altitudes at land are taken by reflection from an artificial horizon. The surface of all fluids are horizontal; but as they are subject to be disturbed by the wind, various contrivances have been invented to render reflecting planes horizontal by means of fluids. The altitude at sea is found by the real horizon, allowing for the dip.

A very full description of all these instruments, with an account of their uses, may be seen in my *Practical Astronomy*.

42. A whirling table is an instrument to show the doctrine of centripetal forces.

43. The eclipsareon is an instrument to show all the phænomena of a solar eclipse.

By this instrument the time and quantity of an eclipse at any place, may be determined to a very considerable degree of accuracy.

44. The figure of the earth, arising from the centrifugal force of its parts in consequence of its rotation about its axis, may be shown by the re-  
R
volution

volution of two brass hoops placed at right angles to each other, and made to revolve about a common diameter.

#### ON THE INEQUALITY OF SOLAR DAYS.

45. By comparing the right ascension of the sun every day at noon with that of a fixed star, it appears that the right ascension of the sun does not increase uniformly.

46. As the earth revolves uniformly about its axis in the same direction in which it revolves about the sun, and the daily increase of the sun's right ascension is not uniform, the solar days, being equal to the time of the earth's rotation + the time of its describing an arc equal to the increase of the sun's right ascension in a solar day, are not all equal.

47. If the motion of the earth in its orbit, or the apparent annual motion of the sun, were uniform and in the equator, the solar days would be all equal.

For the right ascension is measured upon the equator, and therefore in this case its increase would be uniform. Hence if we conceive an imaginary star to move uniformly in the equator with the sun's mean motion in right ascension, the intervals of its transits over the meridian would be all equal. A clock therefore set to 12 when this star is upon the meridian, and adjusted to go 24 hours in that interval, would always agree with the star, that is, show 12 at the time of the transit. Hence it is the same thing, whether we compare solar time with the time measured by this star or by the clock.

48. The sun does not move in the equator, nor does it move uniformly; therefore the solar days are not equal. Hence the inequality of solar days, arises from the inclination of the equator to the ecliptic,



ecliptic, and the unequal angular velocity of the earth in its orbit.

As the solar days are not all equal, a clock adjusted as above described, would not always show 12 when the sun is on the meridian. The time from 12 when the sun is on the meridian, is called the *equation of time*.

The time by the sun is called *apparent*, and that by the clock *true* or *mean* time.

The practical method of computing the equation of time not being given in any books of astronomy, we shall here give the investigation, with an example.

Let *APLS* (fig. 9.) be the ecliptic, *ALv* the equator, *A* the first point of Aries, *P* the sun's apogee, *S* the place of the sun, draw *Sv*, *Pv* perpendicular to *Av*, and take  $Ln = LP$ . When the sun sets out at *P*, let the imaginary star set out at *n* with the sun's mean motion in right ascension or longitude, or at the rate of  $59'.8''$  in a day, and when *n* passes the meridian let the clock be adjusted to 12, in which case *nv* is the equation of time; these are the corresponding positions of the clock and sun as assumed by astronomers. Take  $nm = Ps$ , and when the star comes to *m*, the place of the sun, if it moved uniformly with its mean motion, would be at *s*, but at that time let *S* be the place of the sun. Now as the clock is adjusted to 12 at the time the star at *n* passes the meridian, and as at that time the sun's true place in the equator is *v*, *mv* is the equation of time. Now *s* is the sun's mean place, and as  $An = AP$  and  $nm = Ps$ ,  $\therefore Am = APs$ , consequently  $mv = Av - Am = Av - APs$ . Now let *a* be the mean equinox, and draw *az* perpendicular to *AL*; then  $Am = Az + zm = Aa \times \cos. aAz + zm = \frac{1}{2} Aa + zm$ ; hence  $mv = Av - \frac{1}{2} Aa - zm$ ; now *Av* is the sun's true right ascension, *zm* is the mean right ascension or mean longitude, and *Az* is the equation of the equinoxes in right ascension; hence the equation of time is equal to the difference of the sun's true right ascension, and its mean longitude corrected by the equation of the equinoxes in right ascension. When *Am* is less than *Av*, true time precedes apparent, and when greater, apparent time precedes true; because the earth turns about its axis in the direction *Av*, or order of right ascension, that body whose right ascension is least must come to the meridian first. This rule for computing the equation of time was first given by Dr. MASKELYNE in the *Phil. Transf.* 1764.

As a meridian of the earth, when it leaves *m*, returns to it again in 24 hours, it may be considered, when it leaves that point, as approaching a point at that time  $360^\circ$  from it, and at which it arrives in 24 hours. Hence the relative velocity with which a meridian accedes to or recedes from *m* is at the rate of  $15^\circ$  in an hour. Hence when the meridian passes through *v*, the arc *vm* reduced into time at the rate of  $15^\circ$  in an hour, gives the equation of time at that instant. Hence the equation of time is computed for the instant of apparent noon. Now the time of apparent noon in mean solar time, for which we compute, can only be known by knowing the equation of time. To compute therefore the equation on any day, we must assume the equation

the same as on that day four years before, from which it will differ but very little, and it will give the time of apparent noon, sufficiently accurate for the purpose of computing the equation. If you do not know the equation four years before, compute the equation for noon mean time, and that will give apparent noon accurately enough.

Ex. To find the equation of time on July 1<sup>st</sup>. 1792, for the meridian of *Greenwich*, by *Mayer's* Tables.

The equation on July 1<sup>st</sup>. 1788, was 3'.28'', to be added to apparent noon, to give the corresponding mean time; hence for July 1<sup>st</sup>. 1792, at *oh*. 3'.28'' compute the true longitude.

|                  | Mean Long. ☉    | Long. ☉'s Ap. | N.1.       | N.2. | N.3. | N.4. |
|------------------|-----------------|---------------|------------|------|------|------|
| Epoch for 1752.  | 95.10°.50'.0'.7 | 35.9°.23'.46' | 241        | 227  | 123  | 478  |
| Mean Mot. July 1 | 5.29.23.16,2    | 33            | 163        | 456  | 312  | 27   |
| 3'               | 7,4             |               |            |      |      |      |
| 28''             | 1,1             |               |            |      |      |      |
| Mean Long.       | 3.10.13.25,4    | 3.9.24.19     | 404        | 683  | 435  | 505  |
| Equ. Cen.        | — 1.37,1        | 3.10.13.25,4  |            |      |      |      |
| Equ. ♀ I.        | + 4,5           |               |            |      |      |      |
| ♂ II.            | — 4,7           | 49. 6,4       | Mean Anom. |      |      |      |
| ♀ III.           | + 3,65          |               |            |      |      |      |
| ♂ IV.            | — 0,6           |               |            |      |      |      |
| True Long,       | 3.10.11.51,15   |               |            |      |      |      |

With this true longitude and obliquity 23°.27'.48'',4 of the ecliptic, the true right ascension of the sun is found to be 35.11°.5'.41'',25; also the equation of the equinoxes in longitude = — 0,6; hence

$$\begin{array}{rcl} \text{Mean Long.} & 35.10°.13'.25'',4 \\ \frac{1}{12} \text{ of } - 0,6 & - & 0,55 \end{array}$$

$$\begin{array}{rcl} \text{Mean Long. cor.} & 3.10.13.24,85 \\ \text{Tr. AR.} & 3.11.5.41,25 \end{array}$$

$$\text{Equ.} \quad 52.16,4 \text{ which converted into time gives}$$

3'.29'',1 for the true equation of time; which must be added to apparent to give true time, because the true right ascension is greater than the mean longitude.

#### ON THE SYSTEM OF THE PRIMARY PLANETS.

49. Mercury and Venus revolve about the sun in orbits included within that of the earth's orbit.

For they have all the phenomena which bodies so revolving must necessarily have, that is, two conjunctions, one between the sun and the earth,

earth, for they sometimes appear to pass over the sun's disc as black spots, and the other beyond the sun, when they shine with full faces. Between these situations there is a certain elongation from the sun which they never exceed. From inferior to superior conjunction, Venus is observed to have all the phases of the moon from new to full. Mercury is also observed to have the same phases. The former conjunction is called *inferior* and the latter *superior*. These are called *inferior* planets.

50. If the orbits of these planets were circular, the distance of each from the sun would be to the earth's distance, as the sine of its greatest elongation to radius.

The orbits are not circles but ellipses, having the sun in the focus; for upon that supposition their computed places are always found to agree with their observed places. Their greatest elongations observed in different parts of their orbits are accordingly found to be different. That of Venus being greater than that of Mercury, Mercury must be nearer to the sun than Venus.

51. An inferior planet is direct through superior conjunction and retrograde through inferior, between which situations it is found to be stationary.

52. Venus is a morning star from inferior to superior conjunction; and an evening star from superior to inferior.

53. The earth revolves in an ellipse about the sun, having the sun in one of the foci.

For the computations of the sun's place upon this supposition, allowing for the disturbing forces of the planets, are found to agree with observations.

54. Mars, Jupiter, Saturn and the Georgium Sidus revolve in orbits including that of the earth.

For they appear in opposition, from whence to conjunction they are full orb'd, except Mars which is a little Gibbous in Quadratures. The Georgium Sidus was discovered by Dr. HERSCHEL in 1781. These are called *superior* planets.



55. A superior planet is retrograde in opposition and direct in conjunction, between which situations it is observed to be stationary.

56. If  $a$  be the distance of a planet from the sun, the earth's distance being unity, and  $x$  be the sine of the angle of elongation at the earth when

the planet is stationary, then  $x = \frac{a}{\sqrt{a^2 + a + 1}}$ ,

the orbits being supposed to be circular, and the sun in the center.

57. When a planet is stationary, if  $t$  = the tangent of its elongation, its distance  $a$  from the sun  $= \frac{1}{2}t^2 + t\sqrt{\frac{5}{4}}$ .

Hence if the elongation of a planet be observed when it is stationary, its distance from the sun may be found compared with the earth's distance, upon supposition that the orbits are circular. If the excentricity of the orbit be small, we shall thus get a very near value of the distance.

Ex. In 1782, March 6d. 6h. 14.56" the Georgium Sidus was stationary with 25.28°.49'.27" apparent longitude, and 15'.53" latitude; now the place of the sun at that time was 115.17°.37'; hence the difference of their longitudes was 35.11°.12'.27", consequently the distance of the planet from the sun was 101°.12', being the hypotenuse of a right angled spherical triangle, whose base was 101°.12'.27" and perpendicular 15'.53"; hence  $t = 5.05$ , consequently  $a = 18.4$  which is a near distance of the Georgium Sidus from the sun, the distance of the earth from the sun being unity. If we thus compute the distances of the superior planets, it appears that the order of their distances from the sun are Mars, Jupiter, Saturn, the Georgium Sidus. By computing their distances from the sun in different parts of their orbits, it appears that they are not always at the same distance from the sun; and by assuming an elliptic orbit having the sun in one of the foci, their computed places are found to agree with observation.

58. To determine the periodic time of a planet.

Observe when a planet is in any point of its orbit, and after any number of revolutions observe when it comes to the same point again; then divide that interval of time by the number of revolutions and you get the time of one revolution. The observations of the ancient astronomers are here very useful; for as they have put down the places of  
the

the planets from their observations, by comparing them with the places observed now, we take in a very great number of revolutions, and therefore if we divide the interval of time by the number of revolutions, if a small error be made in the whole time it will affect so much less the time of one revolution.

59. The squares of the periodic times of the planets, have the same proportion as the cubes of their mean distances.

This law was discovered by KEPLER, after having found their periodic times and relative distances. Afterwards Sir I. NEWTON proved, from the laws of gravity, that it must be so.

60. If the planets moved in circular orbits, the time from conjunction to conjunction, or from opposition to opposition of any two, would be equal to the product of their periodic times divided by their difference.

61. The visible enlightened part of a planet varies as the versed sine of the exterior angle at the planet, of a triangle formed by joining the centers of the earth, sun and planet.

62. The geocentric latitude of a planet and its elongation from the sun being known, the heliocentric latitude and longitude may be computed, the ratio of the distances of the earth and planet from the sun being known.

63. Two heliocentric latitudes of a planet, and the difference of longitudes being known, the place of the node and the inclination of the orbit may be computed.

The following table contains the relative distances, periodic times, places of the nodes, inclinations of the orbits to the ecliptic and places of the aphelia of the orbits, of all the planets, according to M. DE LA LANDE.

Mercury

|            | Mean<br>Diff. | Sidereal Revol.  | Node in 1750.  | Inclinat.<br>1780. | Aphelia 1750.  |
|------------|---------------|------------------|----------------|--------------------|----------------|
| Mercury    | 387.10        | 87d.23h.15'.44"  | 15.15°.20'.43" | 7°.0'.0"           | 8s.13°.33'.58" |
| Venus      | 72333         | 224. 16. 49. 11  | 2. 14. 26. 18  | 3.23.35            | 10. 7. 46. 42  |
| Earth      | 100000        | 365. 6. 9. 12    |                |                    | 3. 8. 39. 34   |
| Mars       | 152369        | 686. 23. 30. 36  | 1. 17. 38. 38  | 1.51. 0            | 5. 1. 28. 14   |
| Jupiter    | 520279        | 4332. 14. 27. 11 | 3. 7. 55. 32   | 1.18.56            | 6. 10. 21. 4   |
| Saturn     | 954072        | 10759. 1. 51. 11 | 3. 21. 32. 22  | 2.29.50            | 8.28. 9. 7     |
| Geor. Sid. | 1908180       | 83ys. 150d. 18h. | 3. 12. 33. 31  | 0.46.20            | 11. 17. 6. 44  |

The nodes, inclinations of the orbits and aphelia have a small motion.

Besides these bodies which revolve about the sun, there are others, called *comets*, which revolve in very excentric ellipses. These, as they approach the sun, have generally tails which increase till they come to their perihelia, and then decrease again. They are invisible for the greatest part of their revolutions.

The diameters of the planets, Sun and Moon in English miles are, of the Georgium Sidus 33954, of Saturn 78236, of Jupiter 92414, of Mars 5195, of the Earth 7928, of Venus 7609, of Mercury 3189, of the Sun 877547, and of the Moon 2326.

The mean semidiameter of the sun was here taken 16'.3" according to the Nautical Almanac; and the mean semidiameter of the moon 16'.44",5 according to M. DE LA LANDE.

The Earth revolves about her axis in 23h. 56'.4" mean solar time; Saturn in 12h. 13 $\frac{1}{2}$ '; Jupiter in 9h. 56'; Mars in 24h. 40'; Venus in 23h. 20'; and the Sun in 25d. 10h.

The time of Saturn's rotation is computed from Dr. HERSCHEL's ratio of its diameters, which he makes about 11 : 10. The time of rotation of the other planets, or whether they do revolve about their axes or not, have not yet been determined.

#### ON THE MOON.

64. The moon revolves about the earth.

65. The orbit which the moon describes about the earth is an ellipse, having the earth in one of the foci.

For the computations of the moon's place upon this supposition, allowing for the disturbing force of the sun, agree with observation.

66. The enlightened part of the moon varies as the versed sine of its elongation from the sun.

Hence, as the curve which divides the light from the dark part of the moon appears an ellipse, its phases may at any time be delineated.

68. If



67. If the angular velocity of the moon about the earth were equal to the angular velocity about her axis, the same face would always be turned towards the earth.

68. The angular velocity of the moon about the earth is not uniform, but the angular velocity about her axis is uniform, therefore the same face is not always turned towards the earth.

69. The time of the moon's rotation about her axis is equal to the mean time in her orbit, because we never see the opposite side of the moon.

The consequence of the two last propositions is, that the moon sometimes shows a little more of her eastern, and sometimes of her western limb, and this is called a *libration in longitude*.

70. The moon's axis is not perpendicular to the plane of her orbit, in consequence of which, we see sometimes one pole and sometimes the other; this is called a *libration in latitude*.

The inclination of her axis to her orbit is  $6^{\circ}.49'$ .

71. In north latitudes when the moon's right ascension is nothing, her orbit at the time of her rising makes the least angle with the horizon; therefore the difference of the times of rising on two successive nights is then the least.

In September this happens when the moon is at the full, and this is called the *harvest moon*. The same circumstance takes place every month, but as it does not happen at the time of the full moon, it is not taken notice of. When her AR. =  $6^{\circ}$ , there is the greatest difference of the times of rising. Those signs which rise with the least angle set with the greatest, and the contrary; therefore when there is the least difference in the times of rising, there is the greatest in setting, and the contrary.

The lunar months, and mean distances of the moon at different times of the year, are not all equal, owing to the action of the sun upon it being different. The time of a mean sidereal month was  $27d.7h.43'.11\frac{1}{2}''$  at

at the beginning of this century; but the mean motion is accelerated. The moon's mean distance is  $60\frac{1}{2}$  semidiameters of the earth; and the mean inclination of her orbit is about  $5^{\circ}.8'$ . The nodes are retrograde at the rate of  $19^{\circ}.19'.45''$  in a year of 365d.; and the apogee is progressive at the rate of  $6'.41''.1'''$  in a day.

## ON THE SATELLITES OF JUPITER, SATURN AND THE GEORGIUM SIDUS.

### 72. Jupiter has four fatellites revolving about it.

Their periodic times are  $1d.18h.27'.33''$ ;  $3d.13h.13'.42''$ ;  $7d.3h.42'.33''$  and  $16d.16h.32'.8''$ . They revolve in orbits circular, or very nearly so, except the fourth which Dr. BRADLEY found to be elliptical. Their distances from the center of Jupiter in terms of his semidiameter are 5,965, 9,494, 15,141 and 26,63 according to Sir I. NEWTON. Their motions are subject to considerable irregularities from their mutual attractions. They suffer eclipses like our moon.

### 73. Saturn has seven fatellites revolving about it.

Their periodic times are  $1d.8h.53'.8''$ ,  $9$ ;  $1d.10h.37'.22''$ ,  $9$ ;  $1d.21h.18'.27''$ ;  $2d.17h.41'.22''$ ;  $4d.12h.25'.12''$ ;  $15d.22h.41'.12''$ ;  $79d.7h.49$ ; the two former were determined by Dr. HERSCHEL, who discovered those two fatellites, and the other by Mr. POUND. The distance of the two first from the center of Saturn are  $36'.79''$  and  $29'.67''$  according to Dr. HERSCHEL, and of the others  $43\frac{1}{2}''$ ;  $56''$ ;  $1'.18''$ ;  $3'$ ;  $8'.42\frac{1}{2}''$  according to M. CASSINI, the semidiameter of Saturn being  $20''$ .

### 74. Saturn is also encompassed with a broad flat ring, whose greatest apparent diameter : that of the planet :: 9 : 4.

This, which was supposed to be only one ring, Dr. HERSCHEL has discovered to be two, both lying exactly in the same plane. He has also discovered that it revolves about an axis perpendicular to its plane in  $10h.32'.15''.4$ . Dr. HERSCHEL has also confirmed, what Sir I. NEWTON had before observed, that the 5th. fatellite revolves about its axis; he makes the time of rotation to be  $79d.7h.47'$ .

### 75. The Georgium Sidus has two fatellites revolving about it.

These were discovered by Dr. HERSCHEL; he makes the times of their synodic revolutions to be  $8d.17h.1'.19''.3$  and  $13d.11h.5'.1''.5$ ; and their distances from the planet  $33''.09$  and  $44''.23$  at the mean distance of the planet. Their orbits are nearly perpendicular to the ecliptic.

ON ECLIPSES.

76. The sun is greater than the earth, and the earth is greater than the moon.

77. If tangents be drawn from the sun to the *corresponding* sides of an opaque body, they terminate the umbra; if they be drawn to the *opposite* sides they terminate the penumbra.

Hence if the opaque body be a sphere, the umbra and penumbra will be conical, and the latter will always be an increasing frustrum of a cone; but as the earth and moon are less than the sun, the former will be a cone which terminates. Hence a section of each perpendicular to the axis is a circle, and the penumbra includes the umbra.

The length of the earth's umbra, at the mean distance of the sun, is about 216 semidiameters of the earth, and the length of the moon's umbra about 59.

In eclipses of the moon the shadow is found to be a little greater than what this rule gives, owing to the atmosphere of the earth. MAYER thinks you ought to add as many seconds as the semidiameter contains minutes,

78. An eclipse of the sun happens at the new moon, and an eclipse of the moon at the full moon.

79. There is not an eclipse of the sun at every new moon, nor an eclipse of the moon at every full moon, owing to the plane of the moon's orbit being inclined to the ecliptic.

80. To determine the limits of a lunar eclipse.

The limits of a lunar eclipse are these. At the full moon,

1. If the moon's latitude = the semidiameter of the earth's shadow + the semidiameter of the moon, there will be *no* eclipse.

2. If the latitude be less than that sum, but greater than their difference, there will be a *partial* eclipse.

3. If the latitude be less than their difference, there will be a *total* eclipse.

Cor. The distance of the center of the earth's shadow from the node, or the sun's distance from the opposite node, at the first limit, is about  $12^{\circ}.23'$ ; within which distance therefore the node must be from opposition that there may be an eclipse of the moon.

What is here usually called the moon's latitude is not strictly so, it being



being a perpendicular to the moon's orbit and not to the ecliptic; it differs however but a very little from it, on account of the small angle between them. The same quantity is called the latitude in the next proposition.

### 81. To determine the limits of a solar eclipse.

The limits of a solar eclipse are these. At the new moon,

1. If the moon's latitude = the semidiameter of the earth + the semidiameter of the moon, there will be *no* eclipse.
2. If the latitude be less than that sum, but greater than their difference, there will be a *partial* eclipse.
3. If the latitude be less than their difference, there will be a *total* eclipse.

Cor. The distance of the sun from the node at the first position is about  $16^{\circ}.35'$ ; within which distance therefore the earth must be from the node that there may be an eclipse.

### 82. There are more eclipses of the sun than of the moon; but more eclipses of the moon than of the sun are seen at any given place.

If the nodes of the moon's orbit were fixed, there could not be more than six eclipses in a year, nor less than two; but in consequence of the retrograde motion of the moon's nodes, there may be seven in a year, but this does not affect the other limit.

### TO FIND THE LONGITUDE.

### 83. The longitude may be found by an eclipse of the moon, or of Jupiter's satellites.

For having the hour when they happen at Greenwich, find by prop. 16, 17. the hour when they happen at any other place, and the difference of the times converted into degrees at the rate of  $15^{\circ}$  for an hour gives the difference of longitudes; and the longitude of the place is east or west from Greenwich, according as the hour at the place is before or after that at Greenwich. The eclipses of Jupiter cannot be observed at sea from the unsteadiness of the ship.

### 84. The longitude may also be found by a watch which keeps true time.

For if set to the time at Greenwich, it will always show the time at that place, which compared with the time found at any other place, by taking the sun's or a star's altitude, the difference gives the longitude as before.

### 85. The

85. The longitude may be found by taking the moon's distance from the sun or a fixed star.

Observe the distance, and also the two altitudes, and from thence find the true distance, and also the time from the sun's or star's altitude. The true distance of the moon from the sun or star is put down in the Nautical Almanack for every three hours at Greenwich; hence, by proportion, you may find the time at Greenwich when it is at any other distance. Find therefore the time at Greenwich when it is at the true distance deduced from observation, and the difference between that time and the time at the place of observation gives the difference of longitudes. See all the rules, with an example, in my *Practical Astronomy*, p. 51.

#### ON THE PROGRESSIVE MOTION OF LIGHT, AND THE ABERRATION OF THE FIXED STARS.

86. Light is not instantaneous but progressive.

This was discovered by M. REAUMUR, who observed that an eclipse of Jupiter's satellites began about 16' sooner when Jupiter was in opposition than when in conjunction, which he determined could arise from no other cause than the progressive motion of light, which would have to move over the diameter of the earth's orbit more in the latter case than in the former, before the eclipse would appear to begin.

87. From the progressive motion of light and of the earth in its orbit, the place of every fixed star, measured by an instrument, is forwarder than its true place, reckoned according to the motion of the spectator.

88. The sine of the angle of aberration : the sine of the angle which the line joining the earth and the apparent place of the star makes with the direction of the earth's motion :: the velocity of the earth : the velocity of light.

Cor. As the former angle is observed to be 20" when the latter becomes a right angle, we have, the velocity of the earth : velocity of light ::  $\sin. 20''$  :  $\text{rad.}$  :: 1 : 10314. Hence also this aberration is nothing when the spectator moves directly to or from a star. The place measured by the instrument is called the *apparent place*.

89. Whilst

89. Whilst the earth makes one revolution in its orbit, the curve described by the apparent place of a star upon a plane parallel to the ecliptic is a circle.

The true place of the star in that circle divides that diameter, which, if projected upon the ecliptic, would be perpendicular to the major axis of the earth's orbit, in the same ratio as one of the foci divides that major axis.

90. Every star *appears* to describe an ellipse, whose major axis : minor :: radius : sine of the star's latitude.

The major axis of this ellipse for all stars is  $40''$ , and is always parallel to the ecliptic; the minor axis is perpendicular to it. The path appears an ellipse, because it is the abovementioned circle seen obliquely.

91. If  $m$  and  $n$  be the sine and cosine of the earth's distance from syzygies,  $v$  and  $w$  the sine and cosine of the star's latitude to radius unity; then  $vm \times 20'' =$  the aberration in latitude, and  $\frac{n}{w} \times 20'' =$  the aberration in longitude.

The exact agreement of this theory with observations is a very satisfactory proof of the velocity of the earth, and that light is progressive. In 70 observations on  $\gamma$  *Draconis*, Dr. BRADLEY found but one (and that is noted very dubious on account of clouds) which differed more than  $2''$  from theory, and that did not differ  $3''$ . The theory agreeing thus exactly with observation, without considering the annual parallax, proves that the annual parallax is insensible. Hence Dr. BRADLEY deduced the following conclusions. 1. That the light of all the fixed stars arrives at the earth with the same velocity; for the major axis of the ellipse is the same in all the stars. 2. That unless their distances from us are all equal, which is very improbable, their lights are propagated uniformly to all distances. 3. That light comes from the sun to the earth in  $8'.7\frac{1}{2}''$ , and its velocity : that of the earth ::  $10314 : 1$ . 4. That as the velocity of the star light comes out about a mean of the several velocities found from the eclipses of Jupiter's satellites, we may conclude that the velocity of reflected light is equal to that of direct light. 5. And as it is highly probable that the velocity of light from the sun and fixed stars is the same, it follows that its velocity is not altered by reflection into the same medium,



ON THE PHYSICAL CAUSES OF THE PLANETS  
MOTIONS.

92. All the primary planets are urged by forces tending to the sun.

For by observation they all describe equal areas about the sun in equal times, therefore by Sir I. NEWTON'S PRINCIPIA, prop. 2. sect. 2. lib. 1. they are urged by forces tending to the sun. This is called a *centripetal* force. The projectile force of each planet prevents them from descending to the sun, and this is called a *centrifugal* force. By these two forces the planets are retained in their orbits.

93. All the primary planets describe ellipses about the sun, which is in one of the foci.

For by observation the distance of the apses of their orbits in respect to the sun  $= 180^\circ$ , therefore by the PRINCIPIA, sect. 9. prop. 45. lib. 1. the force varies inversely as the square of their distance from the sun; hence by sect. 3. prop. 13. cor. they describe some conic section having the sun in one of the foci, and that conic section must be an ellipse, otherwise the orbit would not have two apses. Hence the force with which the primary planets are urged towards the sun varies inversely as the square of their distance from the sun.

94. All the secondary planets are urged by forces tending to their respective primaries.

For they all describe areas about their respective primaries proportional to the times, except some small irregularities which may be accounted for from the action of the sun, and from their actions upon each other; hence, as before, they are urged by forces tending to their respective primaries.

95. If the sun did not disturb the motion of the moon, it would revolve in an ellipse about the earth in one of its foci.

The apses of the moon's orbit have a small progressive motion. Now it appears by computation, that this motion is just as much as the sun would cause, if the moon revolved in an ellipse about the earth in one of its foci; therefore without such a disturbing force the moon would so revolve. Hence the moon is urged towards the earth by a force which varies inversely as the square of the distance.

96. All the secondary planets are urged towards their respective primaries by forces which vary inversely

versely as the squares of their distances from the primary.

For they all revolve in circles, or very nearly so, and the periodic times are observed to be in a sesquiplicate ratio of their distances. Hence by the *PRINCIPIA*, lib. 1. sect. 2. prop. 4. cor. 6. the force varies inversely as the square of the distance.

All the deviations of the moon's motion from its motion in an immoveable ellipse, may be accounted for from the action of the sun upon it.

The general principles of these irregularities are explained by Sir I. NEWTON in his *PRINCIPIA*, lib. 1. prop. 66. and its corollaries. In lib. 3. he has computed the principal effects, and shown that they agree with observation. Since his time all the smaller effects have been determined, partly by theory and partly by observation, and tables constructed which will give the place of the moon to a very great degree of accuracy.

97. The earth is a spheroid whose polar diameter is less than the equatorial.

This arises from the centrifugal force of its parts from its rotation about its axis. According to Sir I. NEWTON, in lib. 3. prop. 19. the ratio of the diameters is as 229 : 230, which makes the equatorial exceed the polar diameter by about 34 miles.

98. The precession of the equinoxes arises from the attraction of the sun and moon upon the parts of the earth, exterior to that of a sphere upon the polar diameter.

Sir I. NEWTON, in lib. 1. prop. 66. cor. 20. has given the principles upon which this motion of the earth may be accounted for; but in his investigation of the quantity of the effect in lib. 3. prop. 39. he has fallen into an error. As however the earth is not homogeneous, nor the density of the moon accurately known, it is impossible to compute the true quantity with certainty.

99. The tides arise from the action of the sun and moon.

The general principles of the tides, Sir I. NEWTON has explained in lib. 1. prop. 66. cor. 19; and in lib. 3. prop. 24. *de Syst. Mundi*, he has shown that they will account for all the phenomena.

TABLE I.  
The Refractions of the Heavenly Bodies in Altitude

| App.<br>Alt. | Refrac. | App.<br>Alt. | Refrac. | App.<br>Alt. | Refrac. |
|--------------|---------|--------------|---------|--------------|---------|
| D. M.        | M. S.   | D. M.        | M. S.   | D. M.        | M. S.   |
| 0. 0         | 33. 0   | 3. 20        | 13. 34  | 9. 50        | 5. 20   |
| 0. 5         | 32. 10  | 3. 25        | 13. 20  | 10. 0        | 5. 15   |
| 0. 10        | 31. 22  | 3. 30        | 13. 6   | 10. 15       | 5. 7    |
| 0. 15        | 30. 35  | 3. 40        | 12. 40  | 10. 30       | 5. 0    |
| 0. 20        | 29. 50  | 3. 50        | 12. 15  | 10. 45       | 4. 53   |
| 0. 25        | 29. 6   | 4. 0         | 11. 51  | 11. 0        | 4. 47   |
| 0. 30        | 28. 22  | 4. 10        | 11. 29  | 11. 15       | 4. 40   |
| 0. 35        | 27. 41  | 4. 20        | 11. 8   | 11. 30       | 4. 34   |
| 0. 40        | 27. 0   | 4. 30        | 10. 48  | 11. 45       | 4. 29   |
| 0. 45        | 26. 20  | 4. 40        | 10. 29  | 12. 0        | 4. 23   |
| 0. 50        | 25. 42  | 4. 50        | 10. 11  | 12. 20       | 4. 16   |
| 0. 55        | 25. 5   | 5. 0         | 9. 54   | 12. 40       | 4. 9    |
| 1. 0         | 24. 29  | 5. 10        | 9. 38   | 13. 0        | 4. 3    |
| 1. 5         | 23. 54  | 5. 20        | 9. 23   | 13. 20       | 3. 57   |
| 1. 10        | 23. 20  | 5. 30        | 9. 8    | 13. 40       | 3. 51   |
| 1. 15        | 22. 47  | 5. 40        | 8. 54   | 14. 0        | 3. 45   |
| 1. 20        | 22. 15  | 5. 50        | 8. 41   | 14. 20       | 3. 40   |
| 1. 25        | 21. 44  | 6. 0         | 8. 28   | 14. 40       | 3. 35   |
| 1. 30        | 21. 15  | 6. 10        | 8. 15   | 15. 0        | 3. 30   |
| 1. 35        | 20. 46  | 6. 20        | 8. 3    | 15. 30       | 3. 24   |
| 1. 40        | 20. 18  | 6. 30        | 7. 51   | 16. 0        | 3. 17   |
| 1. 45        | 19. 51  | 6. 40        | 7. 40   | 16. 30       | 3. 10   |
| 1. 50        | 19. 25  | 6. 50        | 7. 30   | 17. 0        | 3. 4    |
| 1. 55        | 19. 0   | 7. 0         | 7. 20   | 17. 30       | 2. 59   |
| 2. 0         | 18. 35  | 7. 10        | 7. 11   | 18. 0        | 2. 54   |
| 2. 5         | 18. 11  | 7. 20        | 7. 2    | 18. 30       | 2. 49   |
| 2. 10        | 17. 48  | 7. 30        | 6. 53   | 19. 0        | 2. 44   |
| 2. 15        | 17. 26  | 7. 40        | 6. 45   | 19. 30       | 2. 39   |
| 2. 20        | 17. 4   | 7. 50        | 6. 37   | 20. 0        | 2. 35   |
| 2. 25        | 16. 44  | 8. 0         | 6. 29   | 20. 30       | 2. 31   |
| 2. 30        | 16. 24  | 8. 10        | 6. 22   | 21. 0        | 2. 27   |
| 2. 35        | 16. 4   | 8. 20        | 6. 15   | 21. 30       | 2. 24   |
| 2. 40        | 15. 45  | 8. 30        | 6. 8    | 22. 0        | 2. 20   |
| 2. 45        | 15. 27  | 8. 40        | 6. 1    | 23. 0        | 2. 14   |
| 2. 50        | 15. 9   | 8. 50        | 5. 55   | 24. 0        | 2. 7    |
| 2. 55        | 14. 52  | 9. 0         | 5. 48   | 25. 0        | 2. 2    |
| 3. 0         | 14. 36  | 9. 10        | 5. 42   | 26. 0        | 1. 56   |
| 3. 5         | 14. 20  | 9. 20        | 5. 36   | 27. 0        | 1. 51   |
| 3. 10        | 14. 4   | 9. 30        | 5. 31   | 28. 0        | 1. 47   |
| 3. 15        | 13. 49  | 9. 40        | 5. 25   | 29. 0        | 1. 42   |



TABLE I. continued.

| App. Alt. |    | Refrac. |    | App. Alt. |    | Refrac. |    | App. Alt. |    | Refrac. |    |
|-----------|----|---------|----|-----------|----|---------|----|-----------|----|---------|----|
| D.        | M. | M.      | S. | D.        | M. | M.      | S. | D.        | M. | M.      | S. |
| 30.       | 0  | 1.      | 38 | 50.       | 0  | 0.      | 48 | 70.       | 0  | 0.      | 21 |
| 31.       | 0  | 1.      | 35 | 51.       | 0  | 0.      | 46 | 71.       | 0  | 0.      | 19 |
| 32.       | 0  | 1.      | 31 | 52.       | 0  | 0.      | 44 | 72.       | 0  | 0.      | 18 |
| 33.       | 0  | 1.      | 28 | 53.       | 0  | 0.      | 43 | 73.       | 0  | 0.      | 17 |
| 34.       | 0  | 1.      | 24 | 54.       | 0  | 0.      | 41 | 74.       | 0  | 0.      | 16 |
| 35.       | 0  | 1.      | 21 | 55.       | 0  | 0.      | 40 | 75.       | 0  | 0.      | 15 |
| 36.       | 0  | 1.      | 18 | 56.       | 0  | 0.      | 38 | 76.       | 0  | 0.      | 14 |
| 37.       | 0  | 1.      | 16 | 57.       | 0  | 0.      | 37 | 77.       | 0  | 0.      | 13 |
| 38.       | 0  | 1.      | 13 | 58.       | 0  | 0.      | 35 | 78.       | 0  | 0.      | 12 |
| 39.       | 0  | 1.      | 10 | 59.       | 0  | 0.      | 34 | 79.       | 0  | 0.      | 11 |
| 40.       | 0  | 1.      | 8  | 60.       | 0  | 0.      | 33 | 80.       | 0  | 0.      | 10 |
| 41.       | 0  | 1.      | 5  | 61.       | 0  | 0.      | 32 | 81.       | 0  | 0.      | 9  |
| 42.       | 0  | 1.      | 3  | 62.       | 0  | 0.      | 30 | 82.       | 0  | 0.      | 8  |
| 43.       | 0  | 1.      | 1  | 63.       | 0  | 0.      | 29 | 83.       | 0  | 0.      | 7  |
| 44.       | 0  | 0.      | 59 | 64.       | 0  | 0.      | 28 | 84.       | 0  | 0.      | 6  |
| 45.       | 0  | 0.      | 57 | 65.       | 0  | 0.      | 26 | 85.       | 0  | 0.      | 5  |
| 46.       | 0  | 0.      | 55 | 66.       | 0  | 0.      | 25 | 86.       | 0  | 0.      | 4  |
| 47.       | 0  | 0.      | 53 | 67.       | 0  | 0.      | 24 | 87.       | 0  | 0.      | 3  |
| 48.       | 0  | 0.      | 51 | 68.       | 0  | 0.      | 23 | 88.       | 0  | 0.      | 2  |
| 49.       | 0  | 0.      | 49 | 69.       | 0  | 0.      | 22 | 89.       | 0  | 0.      | 1  |

TABLE II.  
Dec. Parts of an Hour.

|    |        |    |        |
|----|--------|----|--------|
| 1  | ,01666 | 1  | ,00028 |
| 2  | ,03333 | 2  | ,00056 |
| 3  | ,04999 | 3  | ,00083 |
| 4  | ,06666 | 4  | ,00111 |
| 5  | ,08333 | 5  | ,00139 |
| 6  | ,09999 | 6  | ,00167 |
| 7  | ,11666 | 7  | ,00194 |
| 8  | ,13333 | 8  | ,00222 |
| 9  | ,14999 | 9  | ,00250 |
| 10 | ,16666 | 10 | ,00277 |
| 20 | ,33333 | 20 | ,00556 |
| 30 | ,49999 | 30 | ,00833 |
| 40 | ,66666 | 40 | ,01111 |
| 50 | ,83333 | 50 | ,01388 |

TABLE III.  
For converting Time into Deg.  
Min., and Sec. at the Rate of  
24 Hours for 360°.

| Hou. | Deg. | Min.<br>Sec. | Deg.<br>Min. | Min.<br>Sec. | Dec.<br>o Sec. | Sec. |
|------|------|--------------|--------------|--------------|----------------|------|
| 1    | 15   | 1            | 0.           | 15           | ,1             | 1,5  |
| 2    | 30   | 2            | 0.           | 30           | ,2             | 3,0  |
| 3    | 45   | 3            | 0.           | 45           | ,3             | 4,5  |
| 4    | 60   | 4            | 1.           | 00           | ,4             | 6,0  |
| 5    | 75   | 5            | 1.           | 15           | ,5             | 7,5  |
| 6    | 90   | 6            | 1.           | 30           | ,6             | 9,0  |
| 7    | 105  | 7            | 1.           | 45           | ,7             | 10,5 |
| 8    | 120  | 8            | 2.           | 00           | ,8             | 12,0 |
| 9    | 135  | 9            | 2.           | 15           | ,9             | 13,5 |
| 10   | 150  | 10           | 2.           | 30           |                |      |
| 11   | 165  | 20           | 5.           | 00           |                |      |
| 12   | 180  | 30           | 7.           | 30           |                |      |
| 16   | 240  | 40           | 10.          | 00           |                |      |
| 20   | 300  | 50           | 12.          | 30           |                |      |

**TABLE IV.**  
For converting Deg. Min. and Sec.  
into Time, at the Rate of 360°  
for 24 Hours.

| Deg.<br>Min. | Hou.<br>Min. | Min.<br>Sec. | Deg.<br>Min. | Hou.<br>Min. | Min.<br>Sec. | Sec. | Dec. of<br>Sec. |
|--------------|--------------|--------------|--------------|--------------|--------------|------|-----------------|
| 1            | 0.           | 4            | 30           | 2.           | 00           | 1    | ,067            |
| 2            | 0.           | 8            | 40           | 2.           | 40           | 2    | ,133            |
| 3            | 0.           | 12           | 50           | 3.           | 20           | 3    | ,199            |
| 4            | 0.           | 16           | 60           | 4.           | 00           | 4    | ,266            |
| 5            | 0.           | 20           | 70           | 4.           | 40           | 5    | ,333            |
| 6            | 0.           | 24           | 80           | 5.           | 20           | 6    | ,399            |
| 7            | 0.           | 28           | 90           | 6.           | 00           | 7    | ,466            |
| 8            | 0.           | 32           | 100          | 6.           | 40           | 8    | ,533            |
| 9            | 0.           | 36           | 200          | 13.          | 20           | 9    | ,599            |
| 10           | 0.           | 40           | 300          | 20.          | 00           | 10   | ,666            |
| 20           | 1.           | 20           |              |              |              |      |                 |

**TABLE V.**  
Length of circular Arcs to Rad. = 1.

|    |          |    |          |    |           |
|----|----------|----|----------|----|-----------|
| 1° | ,0174533 | 1' | ,0002909 | 1" | ,00000485 |
| 2  | ,0349066 | 2  | ,0005818 | 2  | ,00000970 |
| 3  | ,0523599 | 3  | ,0008727 | 3  | ,00001454 |
| 4  | ,0698132 | 4  | ,0011636 | 4  | ,00001939 |
| 5  | ,0872665 | 5  | ,0014544 | 5  | ,00002424 |
| 6  | ,1047198 | 6  | ,0017453 | 6  | ,00002909 |
| 7  | ,1221730 | 7  | ,0020362 | 7  | ,00003394 |
| 8  | ,1396263 | 8  | ,0023271 | 8  | ,00003878 |
| 9  | ,1570796 | 9  | ,0026180 | 9  | ,00004363 |
| 10 | ,1745329 | 10 | ,0029089 | 10 | ,00004848 |

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## M A G N E T I S M.

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1. **T**HE earth contains a mineral substance which attracts iron, steel and all ferruginous substances; this is called a *natural magnet* or *loadstone*.

The same substance has the power to communicate its properties to iron, steel and all ferruginous substances; the bodies to which these properties are communicated are called *artificial magnets*. These magnets are also made without the natural magnet.

2. If a magnet be suspended at its center of gravity upon a point, so that it may revolve in an horizontal plane, one end will be constantly turned towards the north pole of the earth and the other towards the south pole. The end directed towards the north pole of the earth is called the *north pole* of the magnet, and the other end the *south pole*. This is called a *compass*.

Every magnet has two poles of this kind. The *north pole* is not directed exactly towards the north pole of the earth; the present variation at London, is about  $23\frac{3}{4}^{\circ}$  towards the west, and is increasing. In the year 1657, the magnet was directed exactly towards the north pole. Before that time it was directed towards the east; but ever since that time the deviation has been increasing towards the west. In different parts of the earth's surface the deviation is different. This deviation from the north is called the *variation* of the compass. The variation at the same place is observed to be different at different times of the year, and on different parts of the day, it being affected by heat and cold.

A great circle in the heavens passing through those two points of the horizon to which the poles of a magnet are directed is called the *magnetic meridian*.

3. If



3. If two magnets be suspended and placed near to each other, the *same* poles will *repel* and the *contrary* will *attract* each other.

The point between the two poles where the magnet has no attraction nor repulsion is called the *magnetic center*. This point is not always exactly in the middle between the two poles.

4. If two magnets be made to swim upon any fluid, the *contrary* poles will *attract* each other and bring them together; and the *same* poles will *repel* and separate them.

5. If one end of a magnet be drawn along a needle, or any bar of iron or steel, several times in the same direction, the needle or bar will become magnetic; and that extremity of the needle or bar which the magnet touched last, acquires a polarity contrary to that of the end of the magnet which was applied.

6. If the magnet be drawn in a contrary direction, it will take away the magnetism which it before gave.

7. A bar of iron or steel not magnetic attracts a magnet as much as the magnet attracts the bar.

8. The magnetic power will be increased in two magnets by letting them remain with their opposite poles together.

Magnetic bars should therefore be always left with the opposite poles laid against each other, or by connecting their opposite poles by a bar of iron; if the like poles be laid together, they will diminish or destroy each other's magnetism.

The magnetic power will be increased in a magnet, by letting a piece of iron remain attached to one or both of its poles. A single magnet should therefore be always thus left.

9. A bar of soft steel acquires magnetism faster and loses it faster than an hard bar.

10. To find whether a bar of iron or steel be magnetic, apply one end to the needle, and if it attract both ends of the needle it is not magnetic; but if it attract one end and repel the other it is magnetic.

11. If a magnetic bar be broken into any two parts, each part becomes a complete magnet having two poles, the ends of each next to where it was broken, acquiring a polarity contrary to the other end.

12. A magnet will attract a piece of iron or steel, although any other body should interpose, unless that body be red hot.

13. The attraction between two magnets begin at a greater distance than between a magnet and a piece of iron; but when in contact the magnet attracts the magnet with less force than it attracts the iron.

It appears from hence, that the attraction varies faster in the latter case than in the former.

14. If the contrary poles of a strong and weak magnet be placed together, it often happens that the weaker will have its poles changed, depending upon their different strengths.

Without attending to this it might appear that the same poles attract each other. But this happens for the same reason that a magnet will give magnetism to any piece of iron; it first destroys the same magnetism and then gives it a contrary one.

15. If a bar or needle be laid in the magnetic  
meri-

meridian and an electrical stroke be sent through it, that end towards the north acquires a north polarity, and the other the south polarity.

16. If a bar of soft iron be kept vertical for some time in these parts of the earth, it becomes magnetic, and the lower end will acquire a north polarity; but in the southern parts of the earth it acquires a south polarity: if you invert the bar its polarity will be instantly reversed.

17. A long bar of iron or steel, or a long piece of wire, suppose 3 or 4 feet, may have several poles, the north and south following one another with a magnetic center between each.

18. An iron bar which in one position will attract one end of a magnet, will when held in another position repel it.

19. If a magnet be suspended by an horizontal axis at its center of gravity, so that it may vibrate in a vertical circle, the north pole will here be depressed and the south pole elevated; in the southern parts of the earth, the contrary takes place; and the dip alters gradually from one hemisphere of the earth to the other.

This instrument is called the *dipping* needle; and, like the position when moving in an horizontal plane, the dip is subject to a variation. The dip at London, at this time is about  $72\frac{1}{2}^{\circ}$ , and from the most accurate observations on that dipping needle belonging to the Royal Society, it appears to diminish about  $15'$  in 4 years. In going from the north to the south, the dip does not alter regularly. As it is extremely difficult to balance the needle accurately, the poles of the needle are generally reversed by a magnet, so that its two ends may dip alternately, and the mean of the two is taken.

The phenomena of the *compass* and *dipping needle*, and of the magnetism acquired by an iron bar in a vertical position, leave no room to doubt



doubt but that the cause exists in the earth. Dr. HALLEY supposed that the earth has within it a large magnetic globe, not fixed within to the external parts, having four magnetic poles, two fixed and two moveable, which will account for all the phenomena of the compass and dipping needle. This would make the variation subject to a constant law, whereas we find casual changes which cannot be accounted for upon this hypothesis. This the Doctor supposes may arise from an unequal and irregular distribution of the magnetical matter. The irregular distribution also of ferruginous matter in the shell may also cause some irregularities. Mr. CAVALLO's opinion is, that the magnetism of the earth arises from the magnetic substances therein contained, and that the magnetic poles may be considered as the centers of the polarities of all the particular aggregates of the magnetic substances; and as these substances are subject to change, the poles will change. Perhaps it may not be easy to conceive how these substances can have changed so materially, as to have caused so great a variation in the poles, the position of the compass having changed from the east towards the west about  $33^{\circ}$  in 200 years. Also the gradual, though not exactly regular, change of variation shows, I think, that it cannot depend upon the accidental changes which may take place in the matter of the earth. Dr. HALLEY first laid down charts of the variation of the compass, drawing curve lines through all those places where the variation is the same. These curve lines never cut one another. If a dipping needle be carried from one end of a magnetic bar to the other, when it stands over the south pole the north end of the needle will be directed perpendicularly to it; as the needle is moved, the dip will grow less, and when it comes to the magnetic center it will stand parallel to the bar; afterwards the south end will dip, and the needle will stand perpendicular to the bar when it is directly over the north pole. A bar thus used is called a *terella*, or little earth, the phenomena being similar to those of carrying a dipping needle from the north to the south. According to Dr. HALLEY's Hypothesis, the pole of the magnet within the earth, which makes our needle dip here has a south polarity, as it attracts that which we call the north pole of the needle; and the contrary pole must have a north polarity. The *aurora borealis* appears to have an effect upon the needle, as it has often been observed to be disturbed when that phenomenon has appeared very strong.

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## E L E C T R I C I T Y.

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1. **I**F a glass tube be rubbed with silk, or if a globe or cylinder be turned about its axis and rub against a cushion covered with silk, sparks and flashes of fire will dart from them. This is called electricity; and the luminous matter is called the electric fluid.

The excitation will be stronger if upon the silk you put some amalgam, usually made with five parts of quicksilver and one of zinc.

The bodies which you can thus excite are called *electrics*; those which you cannot excite are called *non-electrics*. Those bodies which being applied to an excited electric receive and transmit the fluid are called *conductors*; those which will not transmit the fluid are called *non-conductors*. Every *electric* is a *non-conductor*, and every *non-electric* is a *conductor*. The electrics, or non-conductors, are glass, sealing-wax, resins, amber, sulphur, baked wood, silk, &c. &c. The non-electrics, or conductors, are metals, ores, quicksilver, all fluids except air and oils, most saline substances, stony substances, green wood, &c. &c. Amongst these, metals are the best conductors. A body is said to be *insulated* when it is supported by non-conductors only. The electric fluid is generally excited by a cylinder and cushion, and against the cylinder an insulated conductor, called the *prime conductor*, is placed to receive and contain the fluid as it is excited. The cushion is insulated, but that insulation may be taken away by hanging a chain from it to the ground.

2. If an electric tube be excited, it will first attract and then repel light bodies, as small pieces of paper, thread, metal, &c. &c. The conductor will also do the same.

U

3. If

3. If two light pith or cork balls be hung together from a non-conductor, or be insulated in any manner, when brought near the prime conductor they will repel each other.

4. If an insulated conductor be connected with the cushion, and there be two pith or cork balls insulated and electrified by it, these balls repel each other.

5. If one insulated ball be electrified by the prime conductor, and another by the conductor on the cushion, when brought together they will attract each other.

6. If one ball be electrified by a smooth excited glass tube, and the other by an excited cylinder of sealing-wax or rosin, they will attract each other.

7. If one ball be electrified by a smooth and the other by a rough excited glass tube, they will attract each other.

8. If a ball be electrified by any excited body, it will attract a body not electrified.

It appears from these experiments that there are two different electric powers; that from the prime conductor, or smooth glass tube, is called *positive*, and sometimes *vitreous* electricity; that from the cushion, sealing-wax, rosin or rough glass tube is called *negative*, and sometimes *resinous* electricity.

Hence the following properties of electric attraction and repulsion.

1. If two bodies be electrified both positively or both negatively, they repel each other.

2. If one be electrified positively and the other negatively, they will attract each other.

3. A body electrified either positively or negatively will attract a body not electrified.



## HYPOTHESES.

Respecting the theory of electricity there are two different Hypotheses, one that there is only one fluid, and the other that there are two. Dr. FRANKLIN'S Hypothesis is, that there is only one fluid, and this theory depends on the following principles. 1. That all terrestrial bodies are full of the electric fluid. 2. That the electric fluid violently repels itself and attracts all other matter. 3. By exciting an electric the equilibrium of the electric fluid contained in it is destroyed, and one part contains more than its natural quantity, and the other less. 4. Conducting bodies connected with that part which contains more electric fluid than its natural quantity, receive it, and are charged with more than their natural quantity; this is called *positive* electricity; if they be connected with that part which has less than its natural quantity, they part with some of their own fluid, and contain less than their natural quantity; this is called *negative* electricity. 5. When one body positively and another negatively electrified are connected by any conducting substance, the fluid in the body which is positively electrified rushes to that which is negatively electrified, and the equilibrium is restored. These are the principles of *positive* and *negative* electricity. The other Hypothesis is, that there are two distinct fluids; this was first suggested by Mr. DU FAYE, upon his discovery of the different properties of excited glass, and excited resins, sealing-wax, sulphur, &c. The following are the principles of this theory. 1. That the two powers arise from two different fluids which exist together in all bodies. 2. That these fluids are separated in non-electrics, by the excitation of electrics, and from thence they become evident to the senses, they destroying each others effects when united. 3. When separated they rush together again with great violence in consequence of their strong mutual attraction, as soon as they are connected by any conducting substance. These are the principles of *vitreous* and *resinous* electricity. This is the theory of Mr. EELES from the hint of two fluids by Mr. DU FAYE.

## 9. The electric fluid is received from the earth.

For if the rubber be insulated no electricity can be produced.

A jar or phial coated on the inside and outside with tinfoil, except about 2 inches on the top, is called the *Leyden* phial or jar, the uses of which in electricity having been discovered at Leyden, by Mr. MUSCHENBROEK.

10. If a brass wire, with a ball on the top, be put into the Leyden phial, and the ball be applied to the prime conductor when electrified, and the outside be connected with the earth by conductors, the inside will be charged with the positive, and

the outside with the negative electricity; and if a communication be formed by a conducting substance between the inside and the outside, an explosion will be heard, and the phial will be discharged.

The communication may be made by a number of persons taking hold of each others hand, when each person will receive a shock.

If this be explained by the first hypothesis, the inside of the jar is said to have more than its natural quantity of the electric fluid, and the outside as much less than its natural quantity, one always gaining exactly as much as the other loses. If it be explained by the second hypothesis, the inside of the jar is said to have received a certain quantity of the vitreous fluid, and the outside as much of the resinous.

11. If the ball of the phial be applied to the conductor on the cushion, the inside will be charged with the negative and the outside with the positive electricity.

As it is necessary for many experiments to collect a large quantity of this fluid, a great many phials are placed together, having their outside and insides respectively connected, in consequence of which they may all be discharged together. This is called a battery.

12. If the outside of an insulated phial be connected with the prime conductor, and the inside of another phial with the inside of that, the outside of the latter having a communication with the earth, both will be equally charged.

In this case the inside of the second phial has received as much fluid from the inside of the first, as the outside thereof received from the machine.

13. If a phial be insulated, it cannot be charged by applying the knob from the wire within to the conductor.

14. Place an insulated phial so that its knob may be about half an inch from the conductor, and whilst the cylinder is turning, hold a brass knob

knob near the coating of the jar, and it will receive a spark from the coating for every one that passes from the conductor to the knob.

These two last facts prove, according to Dr. FRANKLIN's Hypothesis, that the inside can receive no fluid, unless an equal quantity goes off from the outside.

15. Place the knob of an insulated bottle in contact with the prime conductor, and connect the outside coating with the cushion, and the bottle will be charged with its own electricity, the fire going from the outside round by the cushion into the inside.

16. Hang a small linen thread near the coating of an electrified phial, and touch the wire from the inside, and at every touch the thread will be attracted by the coating.

This Dr. FRANKLIN gave as a proof of his Hypothesis; for as the fire was taken from the inside, the outside drew in an equal quantity by the thread.

17. When a phial is charged, the fluid resides in the glass, and not in the coating.

18. If a phial be insulated, and its knob connected to the prime conductor, and the machine be put in motion, a certain quantity of electric fluid will be added to the inside; for if you touch the outside, a quantity nearly equal to that thrown in comes from it.

This experiment is used by Dr. GREY against Dr. FRANKLIN's Hypothesis, as here was a certain quantity of fluid thrown into the inside without any coming from the outside.

19. If a quire of paper be suspended so that it may vibrate freely, and an electrical charge be sent through  
through



through it, it will give it no motion, but the leaves will be protruded both ways from the middle.

Upon supposition that there is only one fluid, it has appeared extraordinary that it should not give the paper any motion. But, as Mr. ARWOOD observes, such a velocity to the fluid may be assigned as shall give a smaller angular velocity to the paper than any that can be assigned, and we know no limit to the velocity of the electric fluid. But the opposite directions in which the leaves are protruded, tends to strengthen the opinion of two fluids coming in an opposite direction.

20. Insulate a charged bottle, and let a cork ball, suspended by silk, hang against the outside; touch the knob, and the ball flies off electrified with the refinous power.

In this case, says Mr. EELES, can any Franklinian suppose, that it is the return of the positive power to the outside of the glass, that electrifies the ball negatively? He further observes, that it is absurd to suppose that bodies negatively electrified are deprived of their natural share of electricity; it being a contradiction to imagine, that bodies will repel each other the more, the more they are divested of the power of repulsion.

21. If a brass wire and ball be fixed to the end of the conductor, and another ball be presented to it, a crooked spark will pass from one to the other, when the machine is highly charged.

The largest sparks proceed from that end of the conductor which is furthest from the cylinder, and is nearly in proportion to the surface of the conductor.

22. If you present only a pointed wire, the electric fluid will pass to it with a hissing noise and in a continual stream. The same follows, if you present a ball to a point from the conductor. If both be points, the electricity is more readily discharged.

Hence pointed conductors are put upon buildings in order to draw off the electric matter gradually from the clouds, by which they may prevent an accumulation of the fluid in the clouds, and thereby a stroke of lightning may be avoided. If a stroke should happen to fall by the sudden coming of a cloud over charged, the conductor may carry it off

off without its hurting the building. The conductors should be elevated two or three yards above the building, and the more there are put upon different parts of the building, the greater will be the safety.

23. The electric fluid appears a diverging stream from a point electrified positively, and like a small star from a point negatively electrified.

It is inferred from hence, that the fluid goes from the point electrified positively, as a fluid diverging from a point will necessarily put on the appearance observed from the point electrified positively; and that it goes to the point electrified negatively, for as the point is surrounded with the fluid, it must break upon it all round from the same distance, and therefore put on the appearance of a star. This experiment seems to be in favour of only one fluid; for if there were two fluids passing in opposite directions, it would not be easy perhaps to account for the different appearances in the two points.

24. If the discharge of a phial be made through the flame of a candle, the flame will be driven towards that part which is electrified negatively.

25. If the discharge be made through a pith ball, it will be driven towards that part which is electrified negatively.

These two experiments favour the doctrine of a single fluid.

26. Present a thin exhausted flask to the conductor, the flask will be illuminated, and will have the appearance of the aurora borealis.

27. Gunpowder and spirits may be fired by the electric fluid.

28. The electric fluid is communicated through the body of the conductor, and not upon its surface.

29. The electricity which is communicated to a body lies on its surface.

For the quantity which a body receives is observed to be in proportion to its surface, and not to its solid content.

30. Smoke,

30. Smoke, and the vapour of hot water, will conduct the electric fluid.

31. A pane of glass may be electrified on both sides with the same power.

32. Cover two large boards with tinfoil, place them parallel, and connect one insulated with the prime conductor, and the other with the ground. Turn the machine, and the former will be electrified positively and the latter negatively; and the air between them keep the two powers asunder. Thus a plate of air is charged.

33. If there be two different conductors to make a discharge, the discharge will always be made through that which conducts best, every thing else being the same.

34. If the conductors be the same, but one shorter than the other, the discharge will be made through the shortest.

35. If two electric plates be charged, and a communication be formed between the vitreous side of one and the resinous of the other, no discharge will follow, unless a communication be formed between the other two sides at the same time.

Upon this Mr. Atwood has made the following observations. The natural electricity in the atmosphere is frequently discharged in this manner: Two clouds being electrified with opposite powers, the surfaces of the earth immediately under them are likewise electrified with powers contrary to those in the clouds above them; and the moisture of the earth forming a communication between the two contiguous charged surfaces, whenever the two clouds meet, there will follow a discharge, both of the clouds and surfaces on the earth opposed to them. If the earth should be dry, and consequently afford a resistance to the union  
of



of the two electricities accumulated on or under its surface, there will follow an explosion in the earth as well as in the atmosphere, which will produce concussions and other phenomena which have frequently been observed to happen in dry seasons, particularly in those climates which are the most liable to storms of thunder and lightning.

36. If a kite be raised in the air with a proper string, the string is observed to conduct a quantity of the electric fluid from the air.

Mr. CAVALLO deduced the following circumstances from his experiments.

1. The air is always electrified positively, and more in frosty than in warm weather.
2. The presence of the clouds generally lessen the electricity; sometimes it has no effect, and it very seldom increases it a little.
3. When it rains the electricity is generally negative, and very seldom positive.
4. The aurora borealis seems not to effect the electricity.

Dr. FRANKLIN has observed that the clouds are sometimes negatively electrified.

Mr. ACHARD observed the electricity of the atmosphere with a pair of pith balls attached to a resinous rod placed above the roof of an house. This electrometer, from its simplicity, is preferable to any other for discovering merely when any electricity exists in the atmosphere.

#### ON THE ANALOGY AND DIFFERENCE BETWEEN MAGNETISM AND ELECTRICITY.

The power of electricity is of two sorts, positive and negative; bodies possessed of the same sort of electricity repel each other, and those possessed of different sorts attract each other. In magnetism, every magnet has two poles, one of which always stands towards the north, and the other towards the south; the same poles repel each other, and the contrary poles attract each other.

In electricity, when a body in its natural state is brought near to one electrified, it acquires a contrary electricity and becomes attracted by it. In magnetism, when a ferruginous substance is brought near to one pole of a magnet, it acquires a contrary polarity, and becomes attracted by it.

One sort of electricity cannot be produced by itself. In like manner, no body can have only one magnetic pole.

The electric virtue may be retained by *electrics*, but it easily pervades *non-electrics*. The magnetic virtue is retained by ferruginous bodies, but it easily pervades other bodies.

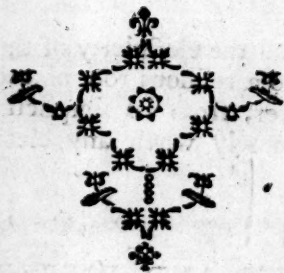
On the contrary, the magnetic power differs from the electric, in that it does not affect the senses with light, smell, taste or noise, as the electric does.

Magnets attract only iron, whereas the electric power attracts bodies of every sort.

The electric virtue resides on the surface of electrified bodies, whereas the magnetic is internal.

A magnet loses nothing of its power by magnetizing other bodies; but an electrified body loses part of its electricity by electrifying other bodies.

Mr. CAVALLO thinks these are the principal points of analogy and difference between magnetism and electricity.



## A D D E N D A.

In *mechanics*, to prop. 17. add this corollary.

Hence the motion in any given direction is not altered by composition.

To the second observation to prop. 21. add,

Hence if the back and two sides be  $B$ ,  $S$ ,  $S'$  respectively, then  $P$ ,  $R$  and  $R'$  will be as  $\frac{B}{p}$ ,  $\frac{S}{r}$  and  $\frac{S'}{r'}$ .

In *optics*, prop. 27. cor. 1. and prop. 57. obser. 1. by the "apparent magnitudes of the image and object" is understood, the angles which they subtend at the eye, and not the apparent magnitudes according to the judgement which the mind forms of them.

## C O R R I G E N D A.

Pag. 13. l. last but 2, for *three-fourths*, read *three-fifths*. Pag. 15. l. 30. for *will*, read *may*. Pag. 25. l. 7. for  $FVv$ , read  $FTv$ . Pag. 136. l. last, for 68, read 67.

The purchasers of my PRACTICAL ASTRONOMY, are requested to make the following corrections.

Pag. 2. l. last, for  $\frac{1}{300}$ , read  $\frac{1}{600}$ . Pag. 4. l. 20, for  $\frac{1}{300}$ , read  $\frac{1}{600}$ . Pag. 40. l. 17. for *latter*, read *former*; and l. 18. for *former*, read *latter*. Pag. 43. l. last but 1, for *at an alt. 44°.22'*, read *in alt. an azim. 44°.22'*; and l. last, for *,2334 of a degree = ,9336'*, read *2', 334 = 9'', 336 in time*. Pag. 47. l. 27. for *below*, read *above*. Pag. 64. l. 24. for  $4''$ , read  $4'$ . Pag. 97. l. 7. for  $a$ , read  $c$ . Pag. 133. l. 19. for *images*, read *limbs*. Pag. 170. l. 20. for 28, read 4. In the *errata*, for Dr. USHER, Professor of Astronomy, at Dublin, read Mr. RAMSDEN.